Southern California Number Theory Day

UC San Diego, March 4-5, 2017

Samit Dasgupta, University of California, Santa Cruz

Title: *On the Gross--Stark Conjecture*

Abstract: In 1980, Gross conjectured a formula for the expected leading term at $s=0$ of the Deligne--Ribet p-adic L-function associated to a totally even character ψ of a totally real field F. The conjecture states that after scaling by $L(\psi \omega^{-1}, 0)$, this value is equal to a p-adic regulator of units in the abelian extension of F cut out by $\psi\omega^{-1}$. In this talk we describe a proof of Gross's conjecture. This is joint work with Mahesh Kakde and Kevin Ventullo.

Roger Heath-Brown, Oxford University and MSRI

Title: *Gaps between smooth numbers*

Abstract: There are many reasons for studying smooth numbers. They provide a toy example, helpful for understanding primes. We will look at the mean square difference between consecutive smooth numbers. The investigation leads to a novel mean value problem for Dirichlet polynomials.

Tasho Kaletha, University of Michigan

Title: *Regular supercuspidal representations and the local Langlands correspondence*

Abstract: Harish-Chandra has given a simple and explicit classification of the discrete series representations of real reductive groups. We will describe a very similar classification that holds for most supercuspidal representations (which we may call regular) of p-adic reductive groups. The analogy runs deeper: there is a remarkable parallel between the characters of regular supercuspidal representations and the characters of discrete series representations of real reductive groups. Guided by this parallel we will give an explicit construction of the local Langlands correspondence for regular supercuspidal representations and discuss some of its properties.

Aaron Pollack, Stanford University

Title: *Twisted orbit parametrizations over the integers*

Abstract: I will explain how to give twisted versions of some of Bhargava's higher composition laws. The key technical idea is a way of relating elements in the open orbit of one prehomogeneous vector space to elements in the minimal nonzero orbit of another.

Nicolas Templier, Cornell University and MSRI

Title: *Mirror symmetry for minuscule flag varieties*

Abstract: We prove cases of Rietsch mirror conjecture that the Dubrovin-Givental quantum connection for projective homogeneous varieties is isomorphic to the pushforward D-module attached to Berenstein-Kazhdan geometric crystals. The idea is to recognize the quantum connection as Galois and the geometric crystal as automorphic. The isomorphism then comes from global rigidity results where a Hecke eigenform is determined by its local ramification. We reveal relations with the works of Gross, Frenkel-Gross, Heinloth-Ngo-Yun and Zhu on Kloosterman sheaves. The talk will keep the algebraic geometry prerequisite knowledge to a

minimum by introducing the above concepts of "mirror" and "crystal" with the examples of CP^1, projective spaces and quadrics. Joint work with Thomas Lam.

Vinayak Vatsal, University of British Columbia

Title: *Test vectors for some ramified representations of GL_2*

Abstract: Let T denote a maximal torus contained in GL_2 of a local field, and let χ denote a character of T.. The study of test vectors for χ-equivariant linear forms on a representation π of GL_2 began with fundamental work of Gross and Prasad, who completely resolved the problem under the assumption that χ and π have disjoint ramification. However, the story is not complete, and one still does not know how to construct natural test vectors in full generality when π and χ are both ramified and T is non-split.

We give a survey of what is known, what we want to know, and we explain why natural test vectors are important for number theory. We also give a simple proof of some results of File, Martin, and Pitale on test vectors when both π and χ are ramified. In particular, we give an explanation of their conditions on highly ramified characters by using some old results of Casselman.

Finally, we analyze the simplest case that remains unresolved, that of depth zero supercuspidal representations. In this case we show that the existence of natural test vectors is equivalent to a bound on certain exponential sums arising from finite Bessel functions.