# Southern California Number Theory Day, UCSD

# May 12, 2018

**Daniel Erman, University of Wisconsin**

**Title:** A probabilistic approach to Noether normalization

**Abstract:** We consider Noether normalization over a finite field from a probabilistic perspective, and use this prove several applications including an effective Noether normalization result over a finite field, and a Noether normalization result over the integers. Our method involves a higher-dimensional variant of Poonen's closed point sieve. This is joint work with Juliette Bruce.

**Stefan Patrikis, University of Utah**

**Title:** Lifting Galois representations with small image

**Abstract:** A basic question in the study of Galois representations, first studied largely in response to Serre's modularity conjecture, is when (or whether) a mod p Galois representation can be lifted to a geometric p-adic representation. Two basic techniques are available: the first comes from applying potential automorphy and automorphy lifting theorems, and gives the most robust results for certain Galois representations valued in classical groups; the second is a purely Galois-theoretic method, originally developed by Ravi Ramakrishna, which applies more readily to general groups. In this talk I will discuss some new techniques in the spirit of Ramakrishna's work. This is joint work with Najmuddin Fakhruddin and Chandrashekhar Khare.

**Gordan Savin, University of Utah**

**Title:** Ext branching problem of Dipendra Prasad.

**Abstract:** The restriction problem from GL(n+1) to GL(n) for p-adic groups, and some variants will be discussed. This is joint work with K. Y. Chan.

**Jacob Tsimerman, University of Toronto**

**Title:** Constructing elliptic curves from Galois representations

**Abstract:** Given a non-isotrivial elliptic curve over an arithmetic surface, one obtains a lisse sheaf of rank two over the surface. This lisse sheaf has a number of straightforward properties: cyclotomic determinant, finite ramification, rational traces of Frobenius, and somewhere not potentially good reduction. We prove that any lisse sheaf of rank two possessing these properties comes from an elliptic curve. This is joint work with Andrew Snowden.

**Don Zagier, Max Planck Institute, Bonn**

**Title:** From knot theory to algebraic units

**Abstract:** In knot theory and more generally the theory of 3-manifolds various "quantum invariants" like the Witten-Reshetikhin-Turaev or the Kashaev invariant have been much studied in recent years, in particular because of the famous "volume conjecture" related to the asymptotic growth of the Kashaev invariant. Rather surprisingly, it transpired a few years ago that these invariants also have very non-trivial number-theoretical properties, including a kind of weak invariance under the modular group SL(2,Z) ("quantum modular forms") and the experimental discovery of the appearance of certain units in cyclotomic extensions as factors in the asymptotic expansions. The talk will report on this and specifically on recent joint work with Frank Calegari and Stavros Garoufalidis that constructs such units in a purely algebraic way starting from elements in algebraic K-theory or in the more elementary "Bloch group". As an unexpected application, this result led to the proof of a well-known conjecture of Nahm on the modularity of certain q-hypergeometric series.