3.2 If a divides b and b divides c then a divides c.

Assume $a|b$ and $b|c$. Then by definition of divides there are integers $k_1, k_2$ such that

$ak_1 = b$ and $bk_2 = c$.

Then $c = bk_2 = ak_1k_2$. Since the product of two integers is also an integer, $a|c$ by definition.

4.2 By contradiction, $n^2$ odd $\Rightarrow$ n odd

Assume for contradiction that there exists an n such that $n^2$ is odd and n is not odd (so even). Since n is even, there is an integer k such that $n = 2k$.

Then $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$ and so $n^2$ is even since $2k^2$ is an integer.

So $n^2$ is even and $n^2$ is odd, a contradiction. Thus, the original ~~statet~~ assumption must be false: there is no n with $n^2$ odd and n even. Thus, $n^2$ odd $\Rightarrow$ n odd.

5.5 $\forall\, a, b \in \mathbb{R}$ and $n \in \mathbb{Z}$, $n \geq 0$, $\sum_{i=0}^{n} (a+ib) = \frac{1}{2}(n+1)(2a+nb)$

The proof will be by induction.

Base case: if $n = 0$ then $\sum_{i=0}^{n} (a+ib) = a + 0b = a$ and $\frac{1}{2}(n+1)(2a+nb) = \frac{1}{2}(1)(2a) = a$

so $\sum_{i=0}^{0} (a+ib) = \frac{1}{2}(0+1)(2a+nb)$ as required.

Inductive step: Suppose $\sum_{i=0}^{k} (a+ib) = \frac{1}{2}(k+1)(2a+kb)$ for some integer k.

I will show that $\sum_{i=0}^{k+1} (a+ib) = \frac{1}{2}(k+2)(2a+(k+1)b)$.

$$
\begin{aligned}
\sum_{i=0}^{k+1} (a+ib) &= \sum_{i=0}^{k} (a+ib) + a + (k+1)b \\
&= \frac{1}{2}(k+1)(2a+kb) + a + (k+1)b \quad \text{(by assumption)} \\
&= \frac{1}{2}\left[(k+1)(2a+kb) + 2a + 2(k+1)b\right] \\
&= \frac{1}{2}\left[2a(k+1) + kb(k+1) + 2a + 2(k+1)b\right] \\
&= \frac{1}{2}\left[2a[(k+1)+1] + b(k+1)[k+2]\right] \\
&= \frac{1}{2}(k+2)(2a+(k+1)b) \text{ as desired.}
\end{aligned}
$$

Thus, by induction, $\sum_{i=0}^{n} (a+ib) = \frac{1}{2}(n+1)(2a+nb)$ for all non-negative integers.