Use residues to derive the integration formulas below:

1. \[ \int_0^\infty \frac{x \sin 2x}{x^2 + 3} \, dx = \frac{\pi}{2} \exp \left(-2\sqrt{3}\right). \]

2. \[ \int_{-\infty}^\infty \frac{x^3 \sin ax}{(x^4 + 4)} \, dx = \pi e^{-a} \cos a \quad (a > 0). \]

3. \[ \int_{-\infty}^\infty \frac{(x+1) \cos x}{x^2 + 4x + 5} \, dx = \frac{\pi}{e} (\sin 2 - \cos 2). \]

4. Exercise 12 of Section 88. Follow the steps outlined in the exercise to evaluate the Fresnel integrals
   \[ \int_0^\infty \cos(x^2) \, dx = \int_0^\infty \sin(x^2) \, dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}. \]

In the following problems, make use of an indented contour of the same kind that we used when we evaluated \( \int_0^\infty \frac{\sin x}{x} \, dx \).

5. Use the function \( f(z) = (e^{iaz} - e^{ibz})/z^2 \) and an indented contour (as in Notes 3) to derive the integration formula
   \[ \int_0^\infty \frac{\cos(ax) - \cos(bx)}{x^2} \, dx = \frac{\pi}{2} (b - a) \quad (a \geq 0, b \geq 0). \]
   Then, using that \( 1 - \cos(2x) = 2 \sin^2 x \), point out how it follows that
   \[ \int_0^\infty \frac{\sin^2 x}{x^2} \, dx = \frac{\pi}{2}. \]

6. Derive the integration formula
   \[ \int_0^\infty \frac{dx}{\sqrt{x} (x^2 + 1)} = \frac{\pi}{\sqrt{2}} \]
   by integrating the function
   \[ f(z) = \frac{z^{-1/2}}{z^2 + 1} = \frac{\exp((-1/2) \log z)}{z^2 + 1} \quad (|z| > 0, -\frac{\pi}{2} < \arg z < \frac{3\pi}{2}) \]
   over an indented contour (as in Notes 3).

7. Derive the integration formula
   \[ \int_0^\infty \frac{(\ln x)^2}{x^2 + 1} \, dx = \frac{\pi^3}{8}. \]
   You will want to use the familiar fact that \( \int_0^\infty \frac{1}{x^2 + 1} \, dx = \frac{\pi}{2} \). You should also obtain, as a by product of the calculation, that \( \int_0^\infty \frac{\ln x}{x^2 + 1} \, dx = 0 \) (note that this does not contradict the example we did in class!).