1. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^5 - 3x^3 + x - 2$. Prove that the equation $f(x) = 0$ has a solution $x$ with $1 < x < 2$.

2. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function whose image consists entirely of rational numbers. Prove that $f$ is a constant function.

   *Hint:* Give a proof by contradiction. If there are $a, b \in [0, 1]$ with $a < b$ and $f(a) \neq f(b)$ then we may apply the use the density of the irrational numbers and then the IVT to get a contradiction.

3. Let $f : D \to \mathbb{R}$ be a Lipschitz function. Prove that $f$ is uniformly continuous.

   The following basic fact was used in the proof of Theorem 3.17.

4. Let $\{a_n\}$ be a sequence of real numbers, and suppose $\{a_n\}$ does not converge to 0 (the sequence $\{a_n\}$ may or may not converge to some other value). Prove that there is a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ with the property that there exists $\epsilon > 0$ such that $|a_{n_k}| \geq \epsilon$ for all $k \in \mathbb{N}$.

   Comment: Note that the converse of Problem 4 also holds, i.e. if $\{a_n\}$ has a subsequence $\{a_{n_k}\}$ with the property that there exists $\epsilon > 0$ such that $|a_{n_k}| \geq \epsilon$ for all $k \in \mathbb{N}$, then the sequence $\{a_n\}$ does not converge to zero.