1. Let \( f : [0, \infty) \to \mathbb{R} \) be given by \( f(x) = \sqrt{x} \). Prove that \( f \) is uniformly continuous.
   
   \text{Hint:} First prove that \(|\sqrt{u} - \sqrt{v}| \leq |\sqrt{u} + \sqrt{v}|\) for all \( u, v \in [0, \infty) \), and then use that this implies \(|\sqrt{u} - \sqrt{v}|^2 \leq |\sqrt{u} - \sqrt{v}| |\sqrt{u} + \sqrt{v}| = |u - v|\).

2. Let \( f : D \to \mathbb{R} \) and \( g : D \to \mathbb{R} \) be uniformly continuous functions.
   
   (a) Give an example showing that \( fg \) need not be uniformly continuous.
   (b) Show that if \( f \) and \( g \) are bounded, then \( fg \) is uniformly continuous.
   
   \text{Hint:} Write \( f(u)g(u) - f(v)g(v) = f(u)[g(u) - g(v)] + g(v)[f(u) - f(v)] \).

3. For each of the following statements, determine whether it is true or false, you do not need to justify your answer.
   
   (a) The function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = 1 \) is uniformly continuous.
   (b) The function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = x \) is uniformly continuous.
   (c) The function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = x^2 \) is uniformly continuous.
   (d) The function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = \sin x \) is uniformly continuous.
   (e) The function \( f : (0, \infty) \to \mathbb{R} \) given by \( f(x) = \frac{1}{x} \) is uniformly continuous.
   (f) The function \( f : [1, \infty) \to \mathbb{R} \) given by \( f(x) = \frac{1}{x} \) is uniformly continuous.
   (g) The function \( f : [-200, 400] \to \mathbb{R} \) given by \( f(x) = x^3 \) is uniformly continuous.
   (h) A function \( f : D \to \mathbb{R} \) is continuous if and only if
   \[
   \forall x_0 \in D \quad \forall \epsilon > 0 \quad \exists \delta > 0 \quad \text{s.t.} \quad |f(x) - f(x_0)| < \epsilon \quad \forall x \in D \text{ with } |x - x_0| < \delta.
   \]
   (i) A function \( f : D \to \mathbb{R} \) is uniformly continuous if and only if
   \[
   \forall \epsilon > 0 \quad \exists \delta > 0 \quad \text{s.t.} \quad \forall x_0 \in D \quad |f(x) - f(x_0)| < \epsilon \quad \forall x \in D \text{ with } |x - x_0| < \delta.
   \]
   (j) A function \( f : D \to \mathbb{R} \) is uniformly continuous if and only if
   \[
   \forall \epsilon > 0 \quad \exists \delta > 0 \quad \text{s.t.} \quad |f(u) - f(v)| < \epsilon \quad \forall u, v \in D \text{ with } |u - v| < \delta.
   \]