Math 142B – Homework Assignment 1  
Due Thursday, April 11  

You do not need to turn in a solution for Problem 5, but give it a shot!

1. Read Chapter 5 of Fitzpatrick, and complete the reading quiz on TritonEd.

2. If $P_1$, $P_2$, and $P_3$ are partitions of $[a, b]$ then a partition $P^*$ is called a common refinement of $P_1$, $P_2$, and $P_3$ if it is a refinement of all three partitions. Find a common refinement of the partitions $P_1 = \{0, \frac{1}{2}, \frac{2}{3}, 1\}$, $P_2 = \{0, \frac{1}{6}, \frac{1}{4}, \frac{1}{2}, 1\}$, and $P_3 = \{0, \frac{2}{3}, \frac{2}{3}, 1\}$ of $[0, 1]$.

3. Let $f : [0, 1] \to \mathbb{R}$ be given by $f(x) = x^2$. Let $n$ be a natural number and let $P_n$ be the regular partition of $[0, 1]$ into $n$ partition intervals (see definition below). Compute $U(f, P_n)$. Simplify your answer using that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

4. Let $f : [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 
1, & \text{if } x \in [0, 1] \setminus \{\frac{1}{2}\} \\
0, & \text{if } x = \frac{1}{2}.
\end{cases}$$

(a) Prove that $\int_0^1 f = 1$.

(b) Prove that $\int_0^1 f = 1$, so that $f$ is integrable and $\int_0^1 f = 1$.

5. Let $f : [a, b] \to \mathbb{R}$ be an integrable function. Prove that if $g : [a, b] \to \mathbb{R}$ differs from $f$ at only a finite number of points in the domain $[a, b]$ then $g$ is integrable and $\int_a^b g = \int_a^b f$.

6. Let $f : [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} 
x, & \text{if } x \in [0, 1] \cap \mathbb{Q} \\
0, & \text{if } x \in [0, 1] \setminus \mathbb{Q}.
\end{cases}$$

(a) Prove that $\int_0^1 f = 0$.

(b) Prove that $\int_0^1 f \geq \frac{1}{2}$.

**Definition:** The partition $P_n = \{x_0, \ldots, x_n\}$ of $[a, b]$ defined by $x_i = a + \frac{i(b-a)}{n}$, for all $i \in \{0, \ldots, n\}$, is called the regular partition of $[a, b]$ into $n$ partition intervals.