Math 184, Fall 2023, Final exam study guide

Practical information:

- **Location is different: Pepper Canyon Hall 109**
- **Time: December 13, 3-6PM**
- This is a closed-book, closed-note exam. Just bring something to write with. *I will provide a sheet with a selected list of formulas, see the last page for what it will contain.*
- Bring your ID.

The last lecture, Dec. 8, will be a review. Same setup as for Midterm 2.

1. Content

The final exam covers the entire course. The new material will receive more emphasis.
The old topics (taken from previous study guides):

- Methods to deal with bijections (sum principle, subtraction principle, product principle, and actually finding bijections)
- Permutations (and variations)
- Words and the problems they apply to
- Choice problems: subsets, multisets, Poker hands
- (Weak) compositions and the problems they apply to
- Set partitions
- Falling factorials: change of basis to powers (in both directions) and their connection to Stirling numbers (of both kinds)
- Cycles in permutations
- Binomial and multinomial theorems: statement, how to simplify sums
- Formal power series: algebra of FPS (addition, multiplication, derivatives, composition), extracting coefficients of terms, general binomial theorem
- Ordinary generating functions: finding simple expressions for generating function given a sequence (either as an explicit formula or as recurrence)
- Linear recurrence relations: using characteristic polynomial to find closed formula, solving some basic inhomogeneous recurrence relations
- Integer partitions: definitions, basic bijections, partition generating functions
- Getting linear recurrence relations from rational functions
- Catalan numbers and how to deal with "quadratic" recurrence relations

New topics not covered on midterms:

- Exponential generating functions:
  - structures, products, and exponential formula.
  - Key examples: permutations, set partitions, labeled graphs.
  - Using derivatives to extract recurrence relations.
- Lagrange inversion formula (Cayley's formula n^{n-2} isn't specifically important, but rather the process behind it is)
- Inclusion-exclusion
- Möbius inversion

Here are the items from the notes that you can safely ignore:

- Theorem 5.1.4: although it's essential in the background for everything that we do, don't worry about memorizing this result.
- Example 6.2.13
- Example 6.2.14
- Remark 7.2.9
- Remark 7.3.3
- Theorem 8.1.8 (and its derivation) and Remark 8.1.9
- Cyclotomic polynomials and Sicherman dice

2. Study advice

Practical information:

- Understanding how to do all of the problems (by yourself, without referencing notes) in the homework is the best way to prepare. All of the problems will be related to something in the homework. This includes Homework 7 even though it is not done for credit.
- Homework and midterm solutions are posted in Canvas, under "files".
- I prefer to emphasize methods and techniques rather than formulas. Of course, remembering formulas is important, but you should think of it as secondary to understanding *how* to solve the homework problems.
- You can review the podcasted lectures through the "media gallery" in Canvas.

Here are some miscellaneous tips:

- Start early. Generally speaking, spreading out your studying across many days is much more effective than cramming the night before.
- When studying problems, it is best to try to do them without looking at any notes and only look at them when you get stuck. This is the best way to develop intuition.
- Form study groups. I've created a `finalexam` channel on the Discord server to discuss studying and practice problems.
- Try to think up your own problems and exchange with friends.
- Take advantage of office hours to clear up any confusions, no matter how minor.

3. A practice exam and more

Here's the final exam from Spring 2023:
https://mathweb.ucsd.edu/~ssam/184/sp23-final.pdf
Here are even more problems from past final exams (including the full Fall 2019 exam):
https://mathweb.ucsd.edu/~ssam/old/23S-184/practice-final.pdf

4. Extra practice from Bóna

If you want additional practice with the material, I have highlighted relevant exercises from the **4th edition** of the textbook.

4.1. Midterm 1.

These have solutions in the book:

- Chapter 3: 1-3, 6-12, 14-15, 18-20, 23
- Chapter 5: 17

The following do not have solutions in the book. I will not provide a solutions manual due to time constraints. However, I am happy to discuss these problems either in office hours or over Discord.

- Chapter 3: 27-37, 41-42, 44-47, 49-50, 52-55
- Chapter 5: 18-21, 23-26, 36-37

4.2. **Midterm 2.** All of these have solutions in the book:

- Chapter 4: 3-4, 18-19, 26-28
- Chapter 5: 6-7, 11
- Chapter 6: 3, 5-6
- Chapter 8: 1-2, 5, 8-9, 11

The following do not have solutions in the book.

- Chapter 4: 41, 43-47, 50
- Chapter 5: 22
- Chapter 8: 25-28, 38-40

4.3. **Post Midterm 2.** Problems with solutions in the book:

- Chapter 7: 3–5, 7–8
- Chapter 8: 21–22

Problems without solutions in the book:

- Chapter 7: 17–20, 28–31
- Chapter 8: 47–48

Formula sheet

Comments: This sheet will be provided for you along with the final exam. Note that not every formula we discuss in lecture is necessary for the exam. In trying to distinguish, be mindful of whether something is important for solving homework problems or is just an auxiliary fact which was derived simply to illustrate a technique.

- The number of k-element multisets of an n-element set is $\binom{n+k-1}{k}$.
- The number of weak compositions of n with k parts is $\binom{n+k-1}{k-1}$.
- The change of basis between powers and falling factorials is
$$x^n = \sum_{k=0}^{n} S(n,k)(x)_k$$
where $S(n,k)$ is the Stirling number of the second kind.
- For $n \geq 0$, we have
$$\sum_{k=0}^{n} (-1)^{n-k} c(n,k) x^k = (x)_n$$
where $c(n,k)$ is the Stirling number of the first kind.
- If d, n are non-negative integers, then
$$\binom{-d}{n} = (-1)^n \binom{d+n-1}{n}.$$
- Lagrange inversion formula: if $G(x)$ is a formal power series with nonzero constant term, then there is a unique formal power series $A(x)$ such that
$$A(x) = xG(A(x)).$$
Furthermore, $A(x)$ has no constant term and for $n \geq 1$, we have
$$[x^n]A(x) = \frac{1}{n}[x^{n-1}](G(x)^n).$$
- Given an alphabet of size k, the number of words of period d is
$$\omega(d) = \sum_{e|d} \mu(d/e) k^e,$$
where μ is the Möbius function, and the number of necklaces of length n is
$$\sum_{d|n} \frac{\omega(d)}{d}.$$