Math 184, Fall 2023
Homework 7

FOR PRACTICE ONLY, DO NOT TURN IN

(1) How many integers between 1 and 1000 are divisible by at least one of 4, 6, or 7?

(2) How many ways are there to list the letters of the following words so that no two consecutive letters are the same?
   (a) PRADEEPKHOSLA
   (b) LAJOLLA

(3) We have $n \geq 2$ married couples ($2n$ people in total).
   (a) How many ways can we have the $2n$ people stand in a line so that no person is standing next to their spouse?
   (b) Same as (a), but replace line by circle (any rotations of a circle are considered the same).

(4) Let A be an alphabet of size n. How many length k words $w_1 \cdots w_k$ are there such that $w_i \neq w_{i+1}$ for $i = 1, \ldots, k-1$ *and* $w_1 \neq w_k$?

(5) Using an alphabet of size k, how many words of period n are there where n is:
   (a) 4
   (b) 6
   (c) 15
   (d) 30

(6) How many necklaces are there of length n using k different colors for the beads where n is:
   (a) 8
   (b) 12
   (c) 30

CHECKING YOUR WORK

This isn't for credit. I'll release solutions around the end of week 10, but it's important that you try to do these problems without looking at answers first since self-checking your final answer is an important step in the learning process.

HINTS

3: Count the "bad" arrangements by tying the couples together with rope. But remember there's more than 1 way to do that.

4: Again, count "bad" arrangements. There's an obvious set to define: something like $A_i = \{w \mid w_i = w_{i+1}\}$. The sizes of the intersections follow a pattern for a while, but it does break down towards the end, so be careful!