Math 184, Midterm 1
Instructor: Steven Sam
April 25, 2023
9:30AM – 10:45AM

Your name:
(Try to use the exact name that is in Gradescope, since it will be automatically matched.)

Student ID:

- **No books, materials, notes, cell phones, calculators, etc.** Consulting other students or any other sources is considered an **academic integrity violation** and will be treated as such.
- Pages will be separated for scanning. **Write your name at the top of each page.** Also, make sure to write **legibly** and **dark enough** and **not too close to the edges of the paper**.
- By default, write your answers only in the space provided. The extra blank sheets can be used for your solution, but **clearly indicate** in the problem if you want the extra sheets to be graded.
- Cross out / erase irrelevant scratch work. If you write incorrect statements without crossing them out, you may lose points. **Make clear what your final answer is.**
- Answers should always have explanations. **You may lose points otherwise.**
- If you finish early, double-check your work and make sure you followed the above instructions. When you're ready, you may turn it in and leave.
- To turn in exam, show your ID and make sure your name is checked off the list.

Good luck!

1. (10 points) Write $f(x) = -2x^2 + 3$ as a linear combination of falling factorials.

2. (10 points) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

where x_1, x_2, x_3, x_4, x_5 are even positive integers?

3. (10 points) How many subsets of [9] contain at least one even number?

4. (15 points) Let n be a positive integer. Below, A, B, C are subsets of $[n]$.

   (a) How many triples (A, B, C) satisfy $A \subseteq B \cap C$?

   (b) How many triples (A, B, C) satisfy $A \subsetneq B \cap C$ (here $\subsetneq$ means "strict subset", so $A \subseteq B \cap C$ **and** $A \neq B \cap C$).

5. For both parts, we're talking about a standard deck of 52 cards (4 suits, 13 values).
   (a) (10 points) How many ways can we choose 6 cards so that they all have the same suit?

   (b) (10 points) How many ways can we choose 8 cards so that we have 4 pairs? For this problem **we do not require that these values are all different** (e.g., two "4 of a kinds" is allowed, along with other possibilities).

6. (15 points) Given non-negative integers k, n, let $M_{k,n}$ be the collection of multisets S of size k of $[n]$ satisfying both properties:
   - Every element appears ≤ 2 times in S, and
   - S has no consecutive values: for all i, if S contains i, then it does not contain $i+1$.

   For example, $M_{3,5}$ consists of the following multisets:

$$\{1,1,3\},\{1,1,4\},\{1,1,5\},\{1,3,3\},\{1,3,5\},\{1,4,4\},\{1,5,5\},$$
$$\{2,2,4\},\{2,2,5\},\{2,4,4\},\{2,5,5\},\{3,3,5\},\{3,5,5\}.$$

   If $k \geq 2$ and $n \geq 2$, prove the following identity:

$$|M_{k,n}| = |M_{k,n-1}| + |M_{k-1,n-2}| + |M_{k-2,n-2}|.$$

Extra scratch paper. If you want this space graded, clearly say so in the problem that you are working on so we know to look here.

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