Math 184, Midterm 2
Instructor: Steven Sam
May 16, 2023
9:30AM – 10:45AM

Your name:

(Try to use the exact name that is in Gradescope, since it will be automatically matched.)

Student ID:

- **No books, materials, notes, cell phones, calculators, etc.** Consulting other students or any other sources is considered an **academic integrity violation** and will be treated as such.
- Pages will be separated for scanning. **Write your name at the top of each page.** Also, make sure to write **legibly** and **dark enough** and **not too close to the edges of the paper**.
- By default, write your answers only in the space provided. The extra blank sheets can be used for your solution, but **clearly indicate** in the problem if you want the extra sheets to be graded.
- Cross out / erase irrelevant scratch work. If you write incorrect statements without crossing them out, you may lose points. **Make clear what your final answer is.**
- Answers should always have explanations. **You may lose points otherwise.**
- If you finish early, double-check your work and make sure you followed the above instructions. When you're ready, you may turn it in and leave.
- To turn in exam, show your ID and make sure your name is checked off the list.

Good luck!

1. (10 points) Let α, β be scalars. What is the coefficient of x^2 in the following formal power series:

$$\frac{(x+\alpha)^{17}}{(1+\beta x)^{13}}$$

2. (10 points) What is the coefficient of $x^2 y^6 z$ in $(2x+y+z)^9$?

3. (10 points) List all of the integer partitions of 6 that have ≤ 3 parts.

4. (20 points) Let (a_n) be the sequence defined by

$$a_0 = 1, \qquad a_1 = 0, \qquad a_n = a_{n-1} + 3a_{n-2} + 2^n \quad (\text{for } n \geq 2).$$

   (a) Define $A(x) = \sum_{n\geq 0} a_n x^n$. Write $A(x)$ as a rational function in x (= polynomial divided by another polynomial). *Tip: most of the points are for setting this up correctly, only worry about simplifying when you're finished with the rest of the exam.*

   (b) Find a *homogeneous* linear recurrence relation satisfied by (a_n). Make sure to state for which n this relation is valid and give all necessary initial conditions. You don't need to find a closed formula.

5. (15 points) Evaluate (and show relevant work)

$$\sum_{\substack{0 \le i \le 15 \\ i \text{ odd}}} i \binom{15}{i} \cdot 6^{i-1} \cdot 2^{15-i}.$$

6. (15 points) For $n \geq 1$, let w_n be the number of set partitions of $[n]$ such that every block has size either 1 or 3.

   (a) Prove that $w_n = w_{n-1} + \binom{n-1}{2} w_{n-3}$ for $n \geq 4$.

   (b) What is w_6? Show work or reasoning.

Extra scratch paper. If you want this space graded, clearly say so in the problem that you are working on so we know to look here.

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