

Name: _____ PID: _____

TA: _____ Sec. No: _____ Sec. Time: _____

Math 10A.
Final Exam
December 8, 2009

Turn off and put away your cell phone.

You may use one page of notes, but no books or other assistance during this exam.

You may leave answers in symbolic form, for example $\sqrt{42}$ or $\ln(6)$.

Read each question carefully, and answer each question completely.

Show all of your work; no credit will be given for unsupported answers.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

#	Points	Score
1	8	
2	4	
3	6	
4	4	
5	4	
6	4	
7	4	
8	6	
9	4	
Σ	44	

1. (8 points) Find the derivative of the given function. Assume that a is a fixed constant.

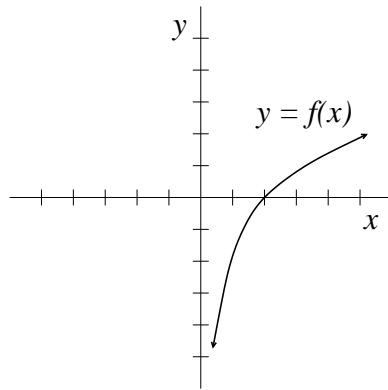
(a) $f(x) = xe^x$

(b) $g(x) = \ln(x^2 + a^2)$

(c) $h(x) = \arctan(2 - x)$

(d) $k(x) = \frac{\sin(ax)}{\cos(x)}$

2. (4 points) Below is the graph of $y = f(x)$:



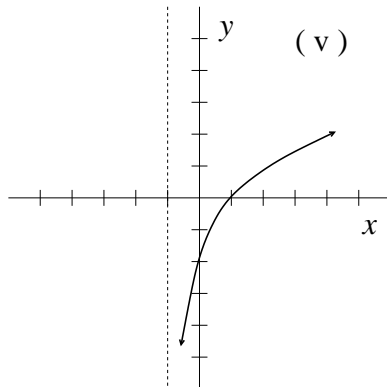
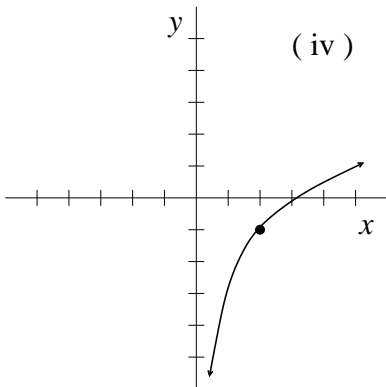
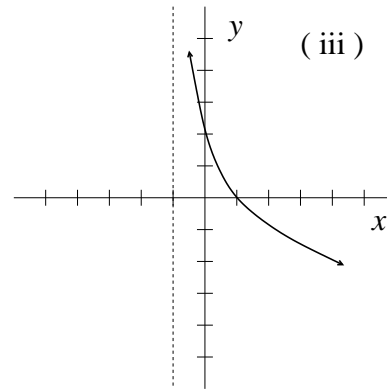
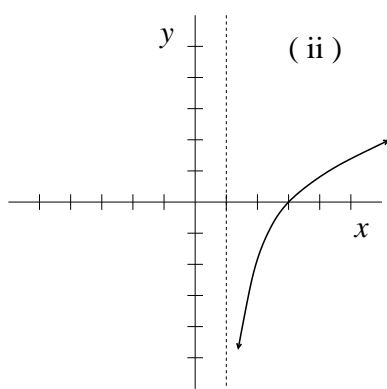
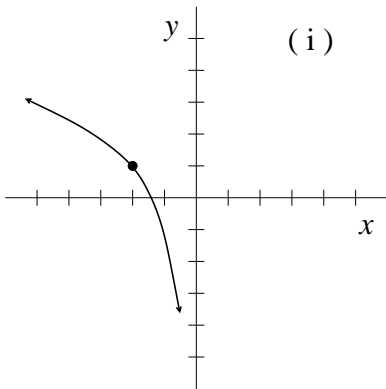
For parts (a)-(d), match each function with the correct graph:

(a) _____ $y = -f(x + 1)$

(b) _____ $y = f(x - 1)$

(c) _____ $y = f(x) - 1$

(d) _____ $y = f(-x) + 1$



3. (6 points) Suppose that $\lim_{x \rightarrow 2} f(x) = 6$ and $\lim_{x \rightarrow 2} g(x) = 4$. Evaluate the given limit, or state that it does not exist:

(a) $\lim_{x \rightarrow 2} (f(x) - 7g(x))$

(b) $\lim_{x \rightarrow 2} x^3 g(x)$

(c) $\lim_{x \rightarrow 2} \frac{f(x)}{x - 2}$

4. (4 points) The position of a car driving on a straight road is given by $s(t) = t^2 - 24 \ln(1 + t)$ for $0 \leq t \leq 3$. Here, t is measured in minutes, and $s(t)$ is measured in meters from the car's starting point.

(a) Find the velocity, $v(t)$, of the car at time t . What are the units of $v(t)$?

(b) Find the time t when the acceleration of the car is equal to 8 m/sec^2 .

5. (4 points) Suppose that f is a function with

$$f'(x) = x(x - 1)^2 \quad \text{and} \quad f''(x) = (3x - 1)(x - 1).$$

(a) On which interval(s) is f increasing?

(b) On which interval(s) is the graph of f concave down?

6. (4 points) Find $\frac{dy}{dx}$ using implicit differentiation:

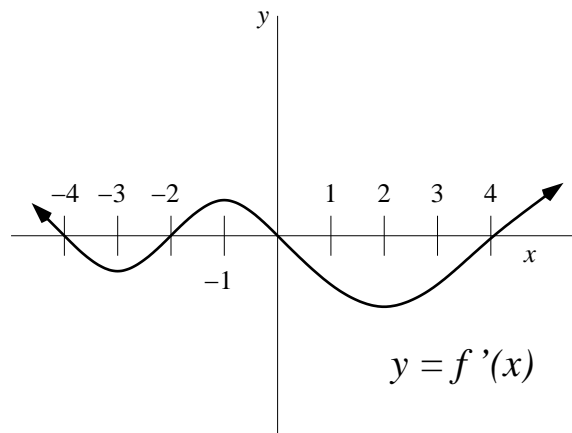
$$\sin(x + y) = y^2 \cos x$$

7. (4 points) Let $g(x) = x^{1/3}$.

(a) Find the equation of the tangent line to the graph of g at the point $(125, 5)$.

(b) Use your answer from part (a) to approximate $(128)^{1/3}$.

8. (6 points) The graph of the *derivative* of a function f is given below:



(a) List all value(s) of x at which $f(x)$ is a local minimum.

(b) List all value(s) of x at which $f(x)$ is a local maximum.

(c) List all value(s) of x at which the graph of f has an inflection point.

9. (4 points) What is the maximum area of an isosceles triangle with two side lengths equal to 5 and one side length equal to $2x$, where $0 \leq x \leq 5$? (Recall that the formula for the area of a triangle is one-half times the length of the base times the height.)

