1. (4 points) The graph of \( y = \cos(t) \) appears below.

On the axes below, sketch the graph of one period of the function \( y = -3 \cos \left( \frac{t}{2} \right) + 1 \).

(1) amplitude = 3, period = \( \frac{2\pi}{\frac{1}{2}} = 4\pi \), maximum = \(+3+1 = 4\), minimum = \(-3+1 = -2\).

(2) \( \cos(t) \rightarrow \cos\left(\frac{t}{2}\right) \rightarrow -3\cos\left(\frac{t}{2}\right) \rightarrow -3\cos\left(\frac{t}{4}\right) + 1 \)
2. (8 points) Use the graphs of the functions $f$ and $g$ which appear below to answer the following questions. If a limit does not exist, please write either “DNE” or “does not exist”.

(a) $g(f(2)) = 0$

$g(2) = 2, \quad g(2) = 0$

(b) $f(g(-1)) = 3$

$g(-1) = 1, \quad f(1) = 3$

(c) List all values of $x$ at which $f$ is not continuous. $-2, 1$

(d) $\lim \limits_{x \to -2^-} f(x) = 1$

(e) $\lim \limits_{x \to -2^+} f(x) = 0$

(f) $\lim \limits_{x \to -2} f(x) = \text{does not exist}$

\[\lim \limits_{x \to -2^+} f(x) = \lim \limits_{x \to -2^-} f(x)\]

(g) $\lim \limits_{x \to 1} \frac{f(x)}{g(x)} = \frac{\lim \limits_{x \to 1} f(x)}{\lim \limits_{x \to 1} g(x)} = \frac{2}{-1} = -2$
3. (4 points) Solve the equation $2e^{qx} = 4^{x+1}$. Please show all of your work.

\[
\ln \left( 2e^{qx} \right) = \ln \left( 4^{x+1} \right)
\]

\[
\ln 2 + \ln e^{qx} = \ln \left( 4^{x+1} \right)
\]

\[
\ln 2 + qx = (x+1) \ln 4
\]

\[
(q - \ln 4)x = \ln 4 - \ln 2 = \ln \frac{4}{2} = \ln 2
\]

\[
x = \frac{\ln 2}{q - \ln 4}
\]
4. (6 points) For each part, find a formula for the indicated quantity:

(a) The number of users of a new iPhone app changes at a constant rate. Every eight hours, there are 32 new users. At midnight, there were 250 users. Find a formula for the number of users \( t \) hours after midnight.

\[
250 + \frac{t}{8} \times 32 = 250 + 4t
\]

*hint*: linear growth, so the function is linear function

250 is initial value

\[
\frac{32}{8} = 4, \quad \text{so 4 new users per hour.}
\]

(b) A woman takes 600 milligrams of ibuprofen. Each hour, the amount of ibuprofen in her system decreases by 8%. Find a formula for the amount of ibuprofen in her system after \( n \) hours.

\[
1 - 8\% = 0.92
\]

So 92% percent of ibuprofen remains each hour.

\[
600 \times 0.92^n
\]
5. (4 points) For this entire problem, let \( g(x) = \frac{10x^2 - 20x}{x^2 - 4x - 5} \)

(a) List the equations of all of the horizontal asymptotes of the graph of \( g \). Please give a brief explanation or show some work which justifies your answer.

\[
\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{10x^2 - 20x}{x^2 - 4x - 5} = \lim_{x \to \infty} \frac{10 - \frac{20}{x}}{1 - \frac{4}{x} - \frac{5}{x^2}} = \frac{10}{1} = 10
\]

\[
\lim_{x \to -\infty} g(x) = \lim_{x \to -\infty} \frac{10x^2 - 20x}{x^2 - 4x - 5} = \frac{10}{1} = 10
\]

Horizontal asymptote of \( g(x) \). \( y = 10 \)

(b) List the equations of all of the vertical asymptotes of the graph of \( g \). Please give a brief explanation or show some work which justifies your answer.

\[
g(x) = \frac{10x(x-2)}{(x+1)(x-5)}
\]

\[
x^2 - 4x - 5 = (x+1)(x-5) = 0 \quad x = -1, 5
\]

Vertical asymptote: \( x = -1, 5 \).
1. 
\[ y = x^3 \quad \Rightarrow \quad y = -x^3 \quad \text{Shift horizontally} \quad \frac{2 \text{ units}}{} \] 
\[ y = -(x-2)^3 \quad \text{Shift vertically} \quad \frac{5 \text{ units}}{5} \delta \]
\[ y = -(x-2)^3 + 5 \]

2. 
(a) \[ f(g(6)) = f(1) = \frac{1}{3} \]
\[ g(6) = 1, \quad f(1) = \frac{1}{3} \]
(b) \[ f(g(\frac{1}{2})) = f(\frac{1}{2}) = \frac{\frac{1}{8}}{\frac{1}{3} + 2} = \frac{1}{7} \]
\[ f(\frac{1}{2}) = \frac{1}{3} \]
(c) \[ f(g(10)) = f(10) = \frac{10}{10 + 2} = \frac{5}{6} \]
\[ g(x) = x - 5 \quad g'(y) = y + 5 \quad g'(5) = 10 \]
(d) \[ f(g(x)) = \frac{g(x)}{g(x) + 2} = \frac{x - 5}{(x - 5) + 2} = \frac{x - 5}{x - 3} \]
3. \[ g(x) = \begin{cases} \frac{x-2}{|x-2|} & x \neq 2 \\ 2 & x = 2 \end{cases} \]

(a) \[ \lim_{x \to 0} g(x) = -2 \quad \lim_{x \to 0^+} g(x) = -2 \quad \lim_{x \to 0^-} g(x) = -2 \]

(b) \[ \lim_{x \to 2^+} g(x) = 2 \quad \lim_{x \to 2^-} g(x) = -2 \quad \lim_{x \to 2} g(x) \text{ does not exist} \]

Since \[ \lim_{x \to 2^+} g(x) \neq \lim_{x \to 2^-} g(x) \]

(c) \[ g(x) \text{ is continuous on } [-1, 1] \quad \text{since } g(x) = -2 \text{ on } [-1, 1] \]

\[ g(x) \text{ is not continuous on } [1, 3], \quad \text{since } \lim_{x \to 2} g(x) \text{ does not exist} \]

4. \[ \lim_{h \to 0} \frac{5}{1+h} - 5 = \lim_{h \to 0} \frac{5 - \frac{1}{1+h} - 1}{h} = \lim_{h \to 0} \frac{5 - \frac{-h}{1+h}}{h} = \lim_{h \to 0} \frac{-5}{1+h} \]

\[ = -5 \]

(b) \[ f(x) = \frac{5}{1+x} \]

\[ f(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{5}{1+h} - 5 \]

Coincide with the formula in (a)