Turn off and put away your cell phone.

No calculators or other electronic devices are allowed during this exam.

You may use one page of notes, but no books or other assistance during this exam. If you violate these instructions or communicate in any way with any other student during this exam, you will receive a zero on the exam, and the zero will not be dropped when calculating your cumulative course average.

Read each question carefully, and answer each question completely.

Show all of your work; no credit will be given for unsupported answers.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

SOLUTIONS
1. (4 points) Evaluate the given integral. Please show all of your work.

\[
\int \frac{\ln(t)}{t^6} \, dt
\]

\[\begin{align*}
\begin{align*}
\text{i} &= \ln(t) \\
\text{d}v &= t^{-6} \, dt \\
\text{d}u &= -\frac{1}{t} \, dt \\
\text{v} &= -\frac{1}{5} \, t^{-5}
\end{align*}
\end{align*}
\]

\[
\int \frac{\ln(t)}{t^6} \, dt = -\frac{1}{5} \ln(t) \cdot t^{-5} + \frac{1}{5} \int t^{-6} \, dt
\]

\[\begin{align*}
\int t^{-6} \, dt &= -\frac{1}{5} \ln(t) \cdot t^{-5} - \frac{1}{25} \cdot t^{-5} + C
\end{align*}\]
2. (4 points) Evaluate the given integral. Please show all of your work.

\[ \int \sin^8(x) \cos^3(x) \, dx \]

\[ \frac{\sin^8(x) \cos^3(x)}{\sin^8(x) \cos^3(x)} \, dx = \int \frac{\sin^8(x) \cos^3(x)}{\sin^8(x) \cos^3(x)} \, dx \]

\[ = \int \sin^8(x) (1 - \sin^2(x)) \cdot \cos(x) \, dx \]

Let \( u = \sin(x) \)
\[ du = \cos(x) \, dx \]

\[ = \int u^8 (1 - u^2) \, du \]

\[ = \int u^8 - u^{10} \, du \]

\[ = \left[ \frac{u^9}{9} - \frac{u^{11}}{11} \right] + C \]

\[ = \left[ \frac{\sin^9(x)}{9} - \frac{\sin^{11}(x)}{11} \right] + C \]
3. (6 points)

(a) Find the partial fraction expansion (PFE) of the rational function:

\[
\frac{4x^2 + 3x + 3}{(3x - 6)(x + 3)^2}
\]

\[
\frac{4x^2 + 3x + 3}{(3x - 6)(x + 3)^2} = \frac{A}{3x - 6} + \frac{B}{x + 3} + \frac{C}{(x + 3)^2}
\]

\((*)\) \quad 4x^2 + 3x + 3 = A(x + 3)^2 + B(3x - 6)(x + 3) + C(3x - 6)

\(x = 2\) in \((*)\):
\[
4(4) + 3(2) + 3 = 25A \\
\Rightarrow 25 = 25A \Rightarrow A = 1
\]

\(x = -3\) in \((*)\):
\[
4(-9) + B(-3) + 3 = C(-9 - 6) \\
\Rightarrow 30 = -15C \Rightarrow C = -2
\]

\(x = 0\) in \((*)\):
\[
3 = 9A - 18B - 6C \\
\Rightarrow 3 = 9(1) - 18B - 6(-2) \\
\Rightarrow -18 = -18B \Rightarrow B = 1
\]

PFE is:
\[
\frac{1}{3x - 6} + \frac{1}{x + 3} - \frac{2}{(x + 3)^2}
\]

Problem continued on next page
(b) Evaluate the integral:

\[
\int \frac{4x^2 + 3x + 3}{(3x - 6)(x + 3)^2} \, dx
\]

\[
= \int \frac{dx}{3x - 6} + \int \frac{dx}{x + 3} - \int \frac{2}{(x+3)^2} \, dx
\]

\[
= \left[ \frac{1}{3} \ln |3x - 6| + \ln |x + 3| \right] - \frac{2}{x + 3} + C
\]

\[
\left( \ln \frac{1}{3} + \ln \frac{1}{2} \right) = - \frac{1}{3} + \frac{1}{2}
\]

\[
\int e^x \frac{1}{e} \, dx = e^x + C
\]

\[
\int e^{x} \frac{1}{e} \, dx = e^x + C
\]

\[
\int e^{x} \frac{1}{e} \, dx = e^x + C
\]

\[
\int e^{x} \frac{1}{e} \, dx = e^x + C
\]
4. (4 points) Determine if the improper integral converges or diverges. If it converges, find its value.

\[ \int_{1}^{\infty} x^2 e^{-x^3} \, dx \]

To find antiderivative:

\[ \int x^2 e^{-x^3} \, dx = -\frac{1}{3} \int e^u \, du \]

\[ u = x^3 \]

\[ du = 3x^2 \, dx \]

\[ \frac{du}{-3} = x^2 \, dx \]

\[ \int \frac{e^u}{3} \, du = -\frac{1}{3} e^u + C \]

\[ \int x^2 e^{-x^3} \, dx = \frac{1}{3} e^{-x^3} + C \]
(a) On the axes provided below, sketch the region enclosed by the graphs of \( y = 2x - 1 \) and \( y = x^2 - 4 \).

\[
2x - 1 = x^2 - 4 \\
\Rightarrow x^2 - 2x - 3 = 0 \\
(x - 3)(x + 1) = 0 \\
\Rightarrow x = 3, x = -1
\]

(b) Find the area of the region enclosed by the graphs of \( y = 2x - 1 \) and \( y = x^2 - 4 \).

\[
\text{Area} = \int_{-1}^{3} (2x - 1) - (x^2 - 4) \, dx
\]

\[
= \int_{-1}^{3} (2x - x^2 + 3) \, dx
\]

\[
= \left[ x^2 - \frac{x^3}{3} + 3x \right]_{-1}^{3}
\]

\[
= 9 - \frac{27}{3} + 9 - \left( 1 + \frac{1}{3} - 3 \right)
\]

\[
= 11 - \frac{1}{3} = \sqrt{10 \frac{2}{3}}
\]
6. (4 points) Find the volume of the solid obtained by rotating the region bounded by
the given curves about the x-axis:

\[ y = \frac{1}{(10-x)^2}, \quad x = 0, \quad x = 8, \quad y = 0 \]

In case you find it helpful, the graph of \( y = \frac{1}{(10-x)^2} \) is provided below.

\[ \text{Sol'n: cross-section} \]
\[ \parallel \text{to} \ x\text{-axis} \]

\[ \Rightarrow \text{Vol solid} = \pi \int_0^8 \frac{1}{(10-x)^4} \, dx \]

\[ = +\frac{\pi}{3} (10-x)^{-3} \bigg|_0^8 = +\frac{\pi}{3} \left( 2^{-3} - 10^{-3} \right) \]

\[ \int \frac{1}{(10-x)^4} \, dx = \int \frac{-1}{u^4} \, du = \int -u^{-4} \, du \]

\[ u = 10-x \]
\[ du = - \, dx \Rightarrow - \, du = dx = \frac{1}{3} (10-x)^{-3} + C \]

\[ \text{Area} = \pi \cdot \frac{1}{(10-x)^4} \]