Problems: Note that there are five problems, so make sure to also look at the second page of this assignment!

(1) As you probably remember from calculus, in addition to helping us understand when a function is increasing or decreasing, derivatives give us information about the shape of the graph of a function (i.e. concavity). Use the theorem from Homework 9 Problem 6 together with the analogous result for a function to be increasing on an interval to explain why the following are true (this is not asking for a formal proof, just an explanation using pictures and slopes of tangent lines):

(a) Explain why if the second derivative $f''$ of a function $f$ is negative on an interval, then the graph of $f$ is concave down on that interval.
Please illustrate your explanation with a sketch of each of the following two cases:
- $f'$ negative and $f''$ negative, what can graph of $f$ look like?
- $f'$ positive and $f''$ negative, what can graph of $f$ look like?

(b) Explain why if the second derivative $f''$ of a function is positive on an interval, then the graph of $f$ is concave up on that interval.
Please illustrate your explanation with a sketch of each of the following two cases:
- $f'$ negative and $f''$ positive, what can graph of $f$ look like?
- $f'$ positive and $f''$ positive, what can graph of $f$ look like?

(2) It’s been raining more than usual in Calculusville this month. On January 10, at $t$ hours after midnight, it was raining at a rate of $\cos(\frac{\pi t^2}{2})$ inches per hour.

(a) What is the rate of rainfall at 1:30 am?
(b) Write an expression that equals the total amount of rain that has fallen between midnight and $x$ hours after midnight on January 10.
(c) It’s 8:15 am, and you want to know how much rain has fallen since midnight. The storm is so strong that your internet is down, so you can’t use Wolfram Alpha and need to make the calculation by hand. Your neighbor Jim tells you that between midnight and 8am, there were a total of 3.15 inches of rainfall. Approximate the total amount of rainfall between midnight and 8:15 am.
(d) Use Wolfram Alpha or something similar to find the actual amount of rainfall between midnight and 8:15 am. Compare your approximation from part (b) with the actual value; what is the percent error?

(3) In Geometryville, the total cost of heating a house for the first $t$ days in the period from February 1 until May 1 equals $\int_0^{t/4} \frac{40}{\sqrt{x+1}} dx$ dollars. Approximately how much are the heating costs per day at the beginning of April?
(4) Maggie and Marla are walking door to door selling Girl Scout cookies. At the moment when Maggie has walked \( u \) miles, she is earning \( \frac{200}{u^2 + 2} \) dollars per mile. Marla started earlier than Maggie and is walking twice as fast as Maggie, so that when Maggie has walked \( u \) miles, Marla has walked \( 2u + 1 \) miles.

(a) Write an expression that equals the total amount of money Maggie has earned when she has walked \( x \) miles.
(b) Write an expression that equals the total amount of money Marla has earned when Maggie has walked \( x \) miles.
(c) What is the rate of change of Maggie’s earnings with respect to the distance that she has walked?
(d) What is the rate of change of Marla’s earnings with respect to the distance Maggie has walked?

(5) (a) The cost of shipping a certain company’s product in a cubic shipping box is given by

\[
c(t) = \int_0^{t^{1/3}} 10\sqrt{1 + x^3} \, dx,
\]

where \( t \) is the length of a side of the box, measured in centimeters. A worker at the company needs to ship a cubic box with a side length of 7.5 cm, but the company’s internet is down, so he can’t use Wolfram Alpha to evaluate \( c(7.5) \). He has a receipt from when he shipped a cubic box with a side length of 8 cm; it cost $32.41. If he knows a little calculus, he can approximate how much it will cost to ship the \( 7.5 \times 7.5 \times 7.5 \) box. What is the approximate cost? Please do this problem without a calculator!

(b) Use Wolfram Alpha or something similar to find the actual value of \( c(7.5) \). Compare your approximation from part (a) with the actual value; what is the percent error?