Problems:

(1) Jack and Jill each run 10 kilometers. They start at the same point, run 5 kilometers up a hill, and return to the starting point by the same route. Jack has a 10 minute head start and runs at the rate of 12 km/hr uphill and 18 km/hr downhill. Jill runs 15 km/hr uphill and 20 km/hr downhill.

(a) How far are they from the top of the hill when they pass going in opposite direction?

(b) What is the distance between Jack and Jill at any given moment during the time they are both running?

(2) A real number $n$ is called **even** if $n = 2k$ for some $k \in \mathbb{Z}$. A real number $n$ is called **odd** if $n = 2k + 1$ for some $k \in \mathbb{Z}$. (Here, the notation $\mathbb{Z}$ means the set of integers.)

(a) Prove that the sum of an even and an odd number is an odd number.

(b) Prove that the difference of two even numbers is an even number.

(c) Prove that the product of two odd numbers is an odd number.

(3) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function whose average rate of change is the same on any interval. Prove that $f$ is a linear function. Please give a proof that is accessible to students who have not had a calculus course.

(4) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a linear function. Prove that the average rate of change of $f$ is the same on any interval.