(1) The following table gives the population (in thousands) of a city:

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>78.52</td>
</tr>
<tr>
<td>1970</td>
<td>85.59</td>
</tr>
<tr>
<td>1980</td>
<td>93.29</td>
</tr>
<tr>
<td>1990</td>
<td>101.69</td>
</tr>
<tr>
<td>2000</td>
<td>110.84</td>
</tr>
<tr>
<td>2010</td>
<td>120.8</td>
</tr>
</tbody>
</table>

Use these data to predict the population of the city in the year 2099. Briefly justify your methods.

(2) A rectangle is expanding so that all four sides are getting longer at the same constant rate (so the rectangle is maintaining a rectangular shape). Let \( m \) and \( n \) be the lengths of the sides of the rectangle at a particular time, and let \( h \) be some number satisfying \( 0 < h < \min(2n, 2m) \).

Consider the side expansion in two cases. The first case is when one side length expands from \( m - h/2 \) to \( m + h/2 \), and the other side length expands from \( n - h/2 \) to \( n + h/2 \). The second case is when one side length expands from \( m \) to \( m + h \), and the other side length expands from \( n \) to \( n + h \).

(a) What are the average rates of change of area with respect to time in these two cases?
(b) What are the average rates of change of the perimeter with respect to in these two cases?
(c) What are the average rates of change of the length of the diagonal with respect to time in these two cases?

(3) At 10:00 am, a car driving east on a straight highway is 20 miles east of Whoville. Suppose the velocity of the car \( t \) hours after 10:00 am is \( v(t) = 60 + 12t^2 \).

(a) Use calculus to find the exact location of the car at 1:00 pm.
(b) Approximate the location of the car at 1:00 pm without using calculus as follows. Imagine that the car has a constant velocity during the first hour of the trip, a constant velocity during the second hour of the trip, and a constant velocity during the third hour of the trip (please use reasonable choices for the velocities.) Compare your approximation to the actual location; what is the percent error?
(c) Approximate the location of the car at 1:00 pm without using calculus as follows. Imagine that the car has a constant velocity during each minute of the trip (please use reasonable choices for the velocities.) Compare your approximation to the actual location; what is the percent error?
(d) Find an expression that approximates the location of the car at 1:00 pm by dividing the time elapsed into \( n \) “subintervals” of equal length and imagining that the car has a constant velocity on each of these subintervals.
(e) What does the value of the expression from part (d) tend towards as \( n \) becomes larger and larger? (In the language of calculus, this part is asking for the limit of the expression from part (d) as \( n \) approaches \( \infty \).)
(4) A runner is training for a marathon. One day, she runs north along a straight path for a total of 18 miles. During the time that she is running, her pace at $t$ hours from the start of her run is $\frac{15}{2} - t$ miles per hour.

(a) Approximate the distance that the runner has travelled after 2 hours without using calculus as follows. Imagine that she is running at a constant pace during each minute of her run, i.e. imagine that she is running at a constant pace during the first minute, that she is running at a constant pace during the second minute, and so on. Please use the given function to make reasonable choices for the constant paces.

(b) The actual distance travelled by the runner after $C$ hours equals the definite integral $\int_0^C (\frac{15}{2} - t) \, dt$. Calculate the actual distance travelled by the runner after 2 hours and then find the percent error in your approximation from part (b). Please round your answer to four decimal places.

(c) Use any method you wish to find out how long it takes the runner to complete her 18 mile run.

(5) (a) Sketch the region in the first quadrant bounded by the graph of $y = \sqrt{x}$, the $y$-axis and the line $y = 5$.

(b) Consider the solid generated by rotating the region from part (a) about the $y$-axis. Find a way to estimate the volume of this solid which is accessible to students who have not had a calculus course.

(c) How close is your estimate to the actual value? How could you improve the accuracy?

Note that the actual volume of the solid equals $\int_0^5 \pi y^4 \, dy$. 