Definition of derivative: Let \( f : \mathbb{R} \rightarrow \mathbb{R} \). \( \frac{df}{dx}(x_0) \) is a number with the property that for any positive number \( \varepsilon \), we can find a positive number \( \delta \) so that if \( 0 < |\Delta x| < \delta \), then
\[
\left| \frac{\Delta f}{\Delta x}(x_0, \Delta x) - \frac{df}{dx}(x_0) \right| < \varepsilon.
\]
Note that if no number has this property, we say that \( f \) is not differentiable at \( x_0 \).

(1) Let \( f(x) = 3x^2 + 2x \).

(a) Sketch the graph of \( f \). Sketch the tangent line to the graph of \( f \) at the point \((-1, f(-1))\).

(b) The slope of the tangent line to the graph of \( f \) at the point \((-1, f(-1))\) is \(-4\). This means that on a small interval around \(-1\), the slopes of the secant lines through \((-1, f(-1))\) and \((-1 + \Delta x, f(-1 + \Delta x))\) should be close to \(-4\). How small does \( |\Delta x| \) need to be to guarantee that \( \frac{\Delta f}{\Delta x}(-1, \Delta x) \) is within 0.01 of \(-4\)?

(c) How small does \( |\Delta x| \) need to be to guarantee that \( \frac{\Delta f}{\Delta x}(-1, \Delta x) \) is within 0.001 of \(-4\)?

(d) Use the definition of the derivative to show that \( \frac{df}{dx}(-1) = -4 \).

(e) Use the definition of the derivative to show that \( \frac{df}{dx}(x_0) = 6x_0 + 2 \).

(2) Let
\[
f(x) = \begin{cases} 
1 & x \leq 1 \\
x & x > 1 
\end{cases}
\]

(a) Sketch the graph of \( f \). Sketch the tangent lines to the graph of \( f \) at the points \((2, f(2))\) and \((-3, f(-3))\).

(b) Use the definition of the derivative to show that \( \frac{df}{dx}(x_0) = 0 \) if \( x_0 < 1 \).

(c) Use the definition of the derivative to show that \( \frac{df}{dx}(x_0) = 1 \) if \( x_0 > 1 \).

(d) Use the definition of the derivative to show that \( \frac{df}{dx}(1) \) does not exist, i.e. show that \( \frac{df}{dx}(1) \neq A \) for any real number \( A \).