Using the $\varepsilon - \delta$ definition of the derivative

(1) Let $f(x) = 2(x-1)^2 + \frac{1}{2}$.

(a) How small does $|\Delta x|$ need to be to guarantee that $\dfrac{\Delta f}{\Delta x}(1, \Delta x)$ is within 0.5 of 0?

(b) How small does $|\Delta x|$ need to be to guarantee that $\dfrac{\Delta f}{\Delta x}(1, \Delta x)$ is within 0.1 of 0?

(c) Given $\varepsilon > 0$, how small does $|\Delta x|$ need to be to guarantee that $\dfrac{\Delta f}{\Delta x}(1, \Delta x)$ is within ε of 0?

(d) Use the $\varepsilon - \delta$ definition of the derivative to show that $\dfrac{df}{dx}(1) = 0$.

(e) Use the $\varepsilon - \delta$ definition of the derivative to show that for any $x_0 \in \mathbb{R}$, $\dfrac{df}{dx}(x_0) = 4(x_0 - 1)$.