Square Expansion Problem

(1) Square Expansion Problem: A square has all sides expanding at the same constant rate. Let $s$ be the side length of this square at some time, and let $h$ be some number satisfying $0 < h < 2s$. Consider the side expansion in two cases: The first case is when the side lengths expand from $s$ to $s + h$, and the second case is when the side lengths expand from $s - h/2$ to $s + h/2$. Before doing any calculation, George and Isaac engaged in an argument about the rate of change of the area in these two cases. George argued that the average rates of change of area with respect to time in these cases will be the same, while Isaac was certain that the average rates of change would be different.

(a) If you were engaged in this debate, before doing any calculation, with whom would you side?

(b) Whose prediction turned out to be correct?

(c) Now compare the average rate of change of the square’s area with respect to time when the side lengths expand from $s$ to $s + h$ to the average rate of change of the square’s area with respect to time when the side lengths expand from $s$ to $s + 2h$. 