(1) Prove that a point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.

(2) Suppose that $AB$ and $BC$ are chords of a circle. Let $m$ be the perpendicular bisector of $AB$, and let $n$ be the perpendicular bisector of $BC$. Prove that $m$ and $n$ intersect in the center of the circle.

(3) Translate one of our geometric approaches to the “Circle Problem One” to algebra as follows. Introduce a coordinate system (an $x$-axis and a $y$-axis) on the plane containing the circle, and suppose that $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ are three points on the circle. Choose any two of the three chords determined by these points, and then find the coordinates of the intersection point of the perpendicular bisectors of these two chords, i.e. the center of the circle.

(4) All of our approaches to the “Circle Problem One” were based on the geometric definition of a circle, namely, a circle with center $C$ and radius $r$ is the set:

$$\{P \text{ in the plane such that } CP = r\}$$

Note that in the above definition, $CP$ denotes the distance from $C$ to $P$, i.e. the length of the segment $CP$. Use this geometric definition to derive an algebraic definition (i.e. equation) of a circle as follows. Introduce a coordinate system (an $x$-axis and a $y$-axis) on the plane so that the point $C$ has coordinates $(m, n)$. Find the equation satisfied by the points $(x, y)$ on the circle.

(5) Prove that a line and a circle intersect in two points or one point or not at all.