(1) What do you think are the points on an ellipse which are closest to its center? What about furthest? In this problem you will prove your conjecture!

(a) Find the points on the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) (\( a^2 > b^2 \)) which are closest to its center. Also find the distance from those points to the center. Make sure to justify/prove your answers.

*Hint:* You could use the equation of the ellipse to write the distance from any point on the ellipse to the center as a function of just one variable \( x \). Then use calculus to minimize the square of the distance function on the interval \([-a, a]\).

Note that the distance is minimized exactly when its square is minimized, but it is easier to take the derivative of the squared distance function.

(b) Find the points on the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) (\( a^2 > b^2 \)) which are furthest from its center. Also find the distance from those points to the center. Make sure to justify/prove your answers.

(2) Kepler’s First Law of Planetary Motion states that the orbit of a planet is an ellipse with the star at one of its foci. Suppose we impose a coordinate system so that the orbit of a certain planet is the ellipse with equation
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad a^2 > b^2
\]
and the star around which the planet orbits is the focus \((-c, 0)\), where \( c^2 = a^2 - b^2 \).

(a) Find the point in the planet’s orbit which is closest to the star; this point is called the **periastron**. Also find the distance from the periastron to the star. Make sure to justify/prove your answers.

(b) Find the point in the planet’s orbit which is furthest from the star; this point is called the **apastron**. Also find the distance from the apastron to the star. Make sure to justify/prove your answers.

(3) Suppose that \( V \) is a point external to a sphere. Prove that if \( P \) and \( Q \) are two points on the sphere so that the lines \( \overrightarrow{VP} \) and \( \overrightarrow{VQ} \) are tangent to the sphere, then \( VP = VQ \). Note: You may use that a tangent line to a sphere is perpendicular to the radius at the point of contact.