(1) Find an equation for an ellipse with eccentricity $\frac{1}{5}$, one focus the point $(24,1)$ and corresponding directrix the line $x = 0$.

(2) When we first defined a cone, we noted that a cone with vertex at the origin, axis the $z$-axis, and semi-angle $\delta$ has equation $x^2 + y^2 = (\tan \delta)z^2$. Use this to prove that (1) if a plane does not contain the vertex of a cone and (2) if the plane is orthogonal to the axis of the cone, then the intersection of the plane and the cone is a circle. Hint: If a plane has properties (1) and (2), then what is the form of its equation?

(3) In class on Thursday, Mac made a great observation about one of our Dandelin spheres. She pointed out that the circle of tangency with the cone did not appear to be a great circle of the sphere but instead a circle of smaller radius. Let’s investigate further in a specific example. Let $C$ be the cone in $\mathbb{R}^3$ with vertex the point $(0,0,2)$, axis the $z$-axis, and semi-angle equal to $\pi/4$. So the equation for $C$ is

$$x^2 + y^2 = \tan(\pi/4)(z - 2)^2.$$  

Let $S$ be the sphere with center $(0,0,0)$ and radius $\sqrt{2}$. So the equation for $S$ is $x^2 + y^2 + z^2 = 2$. Find the intersection of $C$ and $S$.

(4) (a) Let $C$ be the set of points in the plane satisfying the equation $f(x, y) = 0$. Let $C'$ be the set of points obtained by translating the points in $C$ up 5 units and to the right 1 unit. What equation is satisfied by the points in $C'$? Why?

(b) Let $C$ be the set of points in the plane satisfying the equation $f(x, y) = 0$. Let $C'$ be the set of points obtained by translating the points in $C$ down 5 units and to the right 1 unit. What equation is satisfied by the points in $C'$? Why?

(c) Let $C$ be the set of points in the plane satisfying the equation $f(x, y) = 0$. Let $C'$ be the set of points obtained by translating the points in $C$ up 5 units and to the left 1 unit. What equation is satisfied by the points in $C'$? Why?