Math 121B Theorem “Cheat Sheet”

Please print or hand-write a copy of this cheat sheet well in advance of next Tuesday’s midterm. In your exam solutions, you may use any of the following theorems without proof:

• Given a line \( t \) and a point \( P \) on \( t \), there exists exactly one line containing \( P \) which is perpendicular to \( t \).
• Given a line \( s \) and a point \( M \) which is not on \( s \), there exists exactly one line containing \( M \) which is perpendicular to \( s \).
• Vertical angles are congruent.
• If two parallel lines are cut by a transversal, then corresponding angles are congruent.
• Congruence conditions for triangles: SAS, ASA, SSS (note that SSA does NOT guarantee congruence except in the case of two right triangles)
• Pythagorean theorem: Suppose a triangle has leg lengths \( a \), \( b \), \( c \). Then the triangle is a right triangle with the right angle opposite the leg of length \( c \) if and only if \( a^2 + b^2 = c^2 \).
• The measures of the interior angles of a triangle add up to \( \pi \).
• Inscribed Angle Theorem: The measure of an inscribed angle in a circle is half the measure of its intercepted arc.
• A line is tangent to a circle or an ellipse if and only if it intersects the circle or the ellipse in exactly one point.
• In \( \triangle EPF \), if \( K \) is a point on the line segment \( EF \), then \( \overrightarrow{PK} \) is the angle bisector of \( \angle EPF \) if and only if \( \frac{EK}{EP} = \frac{FK}{FP} \).
• Sum and difference formulas for tangents of angles:
  \[
  \tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)} \quad \tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}
  \]
• Suppose an ellipse has foci \( F_1 = (-c, 0) \) and \( F_2 = (c, 0) \) where \( c > 0 \), length of major axis \( 2a \), and equation \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), where \( b^2 = a^2 - c^2 \). Then if \( P = (x_0, y_0) \) is any point on the ellipse:
  \[
  PF_1 = a + \frac{cx_0}{a} \quad \text{and} \quad PF_2 = a - \frac{cx_0}{a}
  \]