Math 121B Midterm Exam Review Outline

**Topics:** Our midterm exam will cover the mathematical topics from class meetings during week one through week four. The relevant homework assignments are HW1, HW2, HW3, and HW4. In order to do well on the exam, make sure you understand how to solve all of the homework problems from the aforementioned assignments as well as all of the in-class problems from weeks one through week four (you can find copies of the in-class problems on the course calendar).

**Theorems and Definitions:** You are responsible for knowing the following definitions as well as the statements of the following theorems together with their proofs. You should be comfortable using the definitions and applying the theorems in contexts as in the homework and in-class problems.

- **Def’n:** Geometric definition of circle
- **Thm:** The perpendicular bisector of any chord of a circle contains the center of the circle (i.e. contains a diameter of the circle).
- **Def’n:** Algebraic definition of circle (i.e. set of points satisfying a certain equation)
- **Thm:** A line intersects a circle at zero, one, or two points.
- **Def’n:** Three characterizations of tangent line to circle at a given point:
  - perceptual: line that intersects the circle only at that point
  - geometric: line through that point which is perpendicular to the radius at that point
  - algebraic: equation of tangent line at that point
- **Def’n:** Definition of circles being orthogonal at an intersection point
- **Thm:** If two circles are orthogonal at a point of intersection, then they intersect at a second point and are also orthogonal at that point. *Note:* You only need to know how to prove orthogonality at the second point of intersection; you don’t need to know how to prove the existence of the second point of intersection.
- **Def’ns:** Both geometric definitions of ellipse:
  - Fix two points $F_1$ and $F_2$ and a number $d > F_1F_2$. Then an ellipse is the set . . .
  - Fix a point $F$, a line $l$ with $F \notin l$, and a number $e$ so that $0 < e < 1$. Then an ellipse is the set . . .
- **Def’n:** Algebraic definition of ellipse (i.e. set of points satisfying a certain equation)
- **Thm:** The foci of an ellipse are equidistant from its center. (You need to prove this geometrically because our derivation of an equation of an ellipse used this result.)
• Thm: The major axis of an ellipse is longer than the minor axis. (You need to prove this geometrically because our derivation of an equation of an ellipse used this result.)

• Thm: The endpoints of the minor axis of an ellipse are equidistant from the foci. (You need to prove this geometrically because our derivation of an equation of an ellipse used this result.)

• Thm: An ellipse is symmetric with respect to both its major axis and its minor axis. (You can prove this both algebraically and geometrically.)

• Thm: Any chord through the center of an ellipse has the center as its midpoint. (We proved this geometrically, but if you wish, you can try to prove it algebraically since our derivation of an equation of an ellipse did not rely on this result.)

• Def’n: Three characterizations of tangent line to ellipse at a given point:
  – perceptual: line that intersects the ellipse only at that point
  – geometric: line that makes equal angles with the focal radii to that point (see HW4, problems 1 and 2)
  – algebraic: equation of tangent line at that point

Other notes: The exam is closed book and closed notes, but you will be provided with the “cheat sheet” which is posted in the announcements section of the course website. You are allowed to use a calculator during the exam, but you cannot use an iPhone or any other device that can connect to the internet.