Conic Section Theorems, Math 121B

**Theorem:** Suppose we have a cone whose semi-angle has measure $\delta$. Let $\Pi$ be a plane which does not contain the vertex of the cone. Let $\gamma$ be the measure of the angle between the plane $\Pi$ and the axis of the cone.

1. If $\gamma = \pi/2$, then the intersection of the cone and the plane is a circle.
2. If $\pi/2 > \gamma > \delta$, then the intersection of the cone and the plane is an ellipse.
3. If $\delta = \gamma$, then the intersection of the cone and the plane is a parabola.
4. If $\delta > \gamma \geq 0$, then the intersection of the cone and the plane is a hyperbola.

**Theorem:** Suppose that the $A$ and $C$ are not both zero, and that the following equation defines a nondegenerate conic section:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

1. If $A = C$, then the equation defines a circle.
2. If $AC > 0$, then the equation defines an ellipse.
3. If $AC = 0$, then the equation defines a parabola.
4. If $AC < 0$, then the equation defines a hyperbola.

**Theorem:** Suppose that $B$ is not zero and that the following equation defines a nondegenerate conic section:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

1. If $4AC - B^2 > 0$, then the equation defines an ellipse.
2. If $4AC - B^2 = 0$, then the equation defines a parabola.
3. If $4AC - B^2 < 0$, then the equation defines a hyperbola.