An ellipse as an intersection of a plane and a cone

**Theorem:** Suppose a plane and a cone are such that the plane does not contain the vertex of the cone and the axis of the cone is not orthogonal to the plane. Then the plane and the cone intersect in an ellipse if and only if

1. The plane intersects every generator of the cone (which forces the intersection of the plane and the cone to be a closed curve).

*Please see Figures One and Two on page two of this document.*

Or, equivalent to (1), the plane and the cone intersect in an ellipse if and only if:

2. The angle between the plane and the axis of the cone is larger than the semi-angle of the cone.

*Please see Figure Three on page three of this document.*

Or, equivalent to (2), the plane and the cone intersect in an ellipse if and only if:

3. The complement of angle between the plane and the axis of the cone is smaller than the base angle of the cone.
In Figure 1, the plane intersects every generator of the cone, and the intersection of the cone and the plane appears to be a closed curve.

In Figure 2, the plane does not intersect every generator of the cone, and the intersection of cone and plane does not appear to be a closed curve.
* This figure illustrates how a plane intersecting every generator of a cone is equivalent to the semi-angle of cone being less than the angle between the plane and axis of cone.

\[ \vec{m} = \text{axis of cone} \]
\[ \vec{v} = \text{normal vector to plane} \]

\[ \delta = \text{measure of semi-angle of cone} \]
\[ \alpha = \text{measure of angle between plane and axis of cone} \]

(Note: \( \alpha = \text{complement of angle} \) between \( \vec{m} \) and \( \vec{v} \))

\[ \varepsilon = \delta \text{ using } \parallel \text{ lines } l \cup n \text{ and transversal } \vec{m} \]

So \( \delta < \alpha \).