Definition of a vector space over $\mathbb{R}$: A vector space (over $\mathbb{R}$) is a nonempty set $V$ of objects called vectors. The vector space comes with two operations on its vectors:

- addition, denoted $+$
- scalar multiplication

The operations must satisfy the following ten axioms for every $\vec{u}, \vec{v}, \vec{w} \in V$ and every $c, k \in \mathbb{R}$:

1. There is a “zero vector”, denoted $\vec{0}$, in $V$ with $\vec{0} + \vec{v} = \vec{v}$.
2. $\vec{u} + \vec{v} \in V$
   Language note: We say that “$V$ is closed under addition” because when you add two vectors in $V$, their sum does not escape from $V$.
3. $c\vec{v} \in V$
   Language note: We say that “$V$ is closed under scalar multiplication” because when you multiply a vector in $V$ by a scalar, the result does not escape from $V$.
4. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
5. $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
6. If $\vec{v}_1 \in V$, then there is a $\vec{v}_2 \in V$ with $\vec{v}_1 + \vec{v}_2 = \vec{0}$.
7. $(c + k)\vec{v} = c\vec{v} + k\vec{v}$
8. $(ck)\vec{v} = c(k\vec{v})$
9. $c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w}$
10. $1\vec{v} = \vec{v}$