1. Suppose \( T : \mathbb{R}^3 \to \mathbb{R}^4 \) is a one-to-one linear transformation. List all of the possible echelon forms of the standard matrix for \( T \). Use the notation of Example 1 in Section 1.2 of your textbook (page 13 of the hard copy of the book). In particular, \( \text{\textbullet} \) denotes a nonzero entry, \( * \) denotes an entry that can be either zero or non-zero, and 0 denotes an entry that must be zero. For example, using this notation, one possible echelon form of a \( 4 \times 4 \) matrix is:

\[
\begin{bmatrix}
\text{\textbullet} & * & * & * \\
0 & \text{\textbullet} & * & * \\
0 & 0 & \text{\textbullet} & * \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

This isn’t one of the answers for this problem, because the dimensions are wrong, but your answer should be a list of matrices of this nature.

2. Suppose \( T : \mathbb{R}^4 \to \mathbb{R}^3 \) is an onto linear transformation. List all of the possible echelon forms of the standard matrix for \( T \). Use the notation of problem one above.

3. If a linear transformation \( T : \mathbb{R}^n \to \mathbb{R}^m \) is onto, what must be true about \( m \) and \( n \)? i.e. can you give a relation between \( m \) and \( n \)?
4. If a linear transformation \( T : \mathbb{R}^n \to \mathbb{R}^m \) is one-to-one, what must be true about \( m \) and \( n \)? i.e. can you give a relation between \( m \) and \( n \)?

5. If the given statement is true, write “True”. If the given statement is false, write “False” and explain why it is false using complete sentences with proper grammar and punctuation. Note that \( A, B, \) and \( C \) denote arbitrary matrices where the indicated sums and products are defined.

(a) Each column of \( AB \) is a linear combination of the columns of \( B \) using weights from the corresponding column of \( A \), e.g. the first column of \( AB \) is the linear combination of the columns of \( B \) using weights from the first column of \( A \), etc.

(b) \( AB + AC = A(B + C) \)

(c) \( A^T + B^T = (A + B)^T \)

(d) \( (AB)^T = A^T B^T \)