Math 18 Written Homework Five
Note that this homework is three pages long!

1. Construct a geometric figure that illustrates why a line in $\mathbb{R}^2$ which does not pass through the origin is not closed under vector addition. Note that here the “vectors” are the points on your line.

2. Let $S$ be the set of all polynomials of the form $p(t) = at^2$, where $a$ is a real number. Is $S$ a subspace of $\mathbb{P}_2$? Carefully justify your answer. (Recall from class that $\mathbb{P}_2$ is the set of all polynomials of degree less than or equal to two).

3. Let $H$ be the set of all polynomials of the form $p(t) = k + t^3$, where $k$ is a real number. Is $H$ a subspace of $\mathbb{P}_3$? Carefully justify your answer. (Recall from class that $\mathbb{P}_3$ is the set of all polynomials of degree less than or equal to three).
4. Let $K$ be the set of all matrices of the form
\[
\begin{bmatrix}
a & b \\
0 & d
\end{bmatrix}
\]. Is $K$ a subspace of the set of all $2 \times 2$ matrices? Carefully justify your answer.

5. Let $A$ be an $m \times n$ matrix. Show that $\text{Col} \ A$ is a subspace of $\mathbb{R}^m$ by demonstrating that the zero vector is in $\text{Col} \ A$, that $\text{Col} \ A$ is closed under addition, and that $\text{Col} \ A$ is closed under scalar multiplication.

6. Let $A$ be an $m \times n$ matrix. Show that $\text{Nul} \ A$ is a subspace of $\mathbb{R}^n$ by demonstrating that the zero vector is in $\text{Nul} \ A$, that $\text{Nul} \ A$ is closed under addition, and that $\text{Nul} \ A$ is closed under scalar multiplication.
7. Let \( S = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p \} \) be a set of \( p \) vectors in \( \mathbb{R}^n \) with \( p < n \). Show that \( S \) cannot be a basis for \( \mathbb{R}^n \).

8. Let \( S = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p \} \) be a set of \( p \) vectors in \( \mathbb{R}^n \) with \( p > n \). Show that \( S \) cannot be a basis for \( \mathbb{R}^n \).

9. If the given statement is true, write “True” below the statement. If the given statement is false, write “False” below the statement. No explanation is required.

   (a) \( \mathbb{R}^2 \) is a subspace of \( \mathbb{R}^3 \).

   (b) A subset \( H \) of a vector space \( V \) is a subspace of \( V \) if the zero vector is in \( H \) and if \( H \) is closed under addition.

   (c) The null space of a matrix \( A \) is the set of solutions of the equation \( A\vec{x} = \vec{0} \).

   (d) The null space of an \( m \times n \) matrix is in \( \mathbb{R}^m \).

   (e) If \( A \) is an \( m \times n \) matrix and the equation \( A\vec{x} = \vec{b} \) is consistent for some \( \vec{b} \) in \( \mathbb{R}^m \), then \( \text{Col} \ A \) is \( \mathbb{R}^m \).

   (f) If \( A \) is a matrix, then \( \text{Col} \ A \) is the set of all vectors that can be written in the form \( A\vec{x} \) for some \( \vec{x} \).

   (g) A linearly independent set in a subspace \( H \) is a basis for \( H \).

   (h) If a finite set \( S \) of nonzero vectors spans a vector space \( V \), then some subset of \( S \) is a basis for \( V \).

   (i) A basis for a subspace \( H \) must be a linearly independent set.

   (j) If \( B \) is an echelon form of a matrix \( A \), then the pivot columns of \( B \) are a basis for \( \text{Col} \ A \).