1. If the given statement is true, write “True” below the statement. If the given statement is false, write “False” below the statement. No justification is required.

(a) \( \mathbb{R}^2 \) is a two-dimensional subspace of \( \mathbb{R}^3 \).

(b) The number of basic variables of a matrix \( A \) equals the dimension of \( \text{Nul} \ A \).

(c) The only three-dimensional subspace of \( \mathbb{R}^3 \) is \( \mathbb{R}^3 \) itself.

(d) \( \mathbb{R}^5 \) has exactly one one-dimensional subspace.

(e) Suppose \( V \) is a nonzero finite-dimensional vector space. If there exists a set \( \{\vec{v}_1, \ldots, \vec{v}_p\} \) which spans \( V \), then \( \dim V \leq p \).

(f) Suppose \( V \) is a nonzero finite-dimensional vector space. If \( \dim V = p \), then there exists a set of \( p + 1 \) vectors that spans \( V \).

(g) If \( B \) is any echelon form of a given matrix \( A \), then the pivot columns of \( B \) form a basis for the column space of \( A \).

(h) Row operations preserve the linear dependence relations among the rows of a matrix. In particular, the linear dependence relations among the rows of a matrix are exactly the same as the linear dependence relations among the rows of the RREF of the matrix.

(i) Suppose \( V \) is a nonzero finite-dimensional vector space. If there exists a linearly independent set \( \{\vec{v}_1, \ldots, \vec{v}_p\} \) in \( V \), then \( \dim V \geq p \).

(j) The row space of a matrix \( A \) is the same as the column space of the matrix \( A^T \).

(k) If a matrix \( B \) is obtained from a matrix \( A \) by a sequence of row operations, then the row space of \( B \) is the same as the row space of \( A \).

(l) If a matrix \( B \) is obtained from a matrix \( A \) by a sequence of row operations, then the column space of \( B \) is the same as the column space of \( A \).
2. Suppose $A$ is a $6 \times 11$ nonzero matrix. What is the smallest possible dimension of $\text{Nul} \ A$? What is the largest possible dimension of $\text{Nul} \ A$? Please explain the reasons for your answers using complete sentences.

3. Suppose a nonhomogeneous system of nine linear equations in twelve unknowns has a solution, with three free variables. Is it possible to change some constants on the right hand sides of the equations to create a new inconsistent system? Please explain the reason for your answer using complete sentences.

4. Suppose a nonhomogeneous system of sixteen linear equations in seventeen unknowns has a solution for all possible constants on the right hand sides of the equations. Is it possible to find two nonzero solutions of the associated homogeneous system that are not multiples of one another? Please explain the reason for your answer using complete sentences.
5. A scientist solves a nonhomogeneous system of eighteen linear equations in twenty unknowns and finds that three of the unknowns are free variables. Can the scientist be certain that, if the right sides of the equations are changed, then the new nonhomogeneous system will still have a solution? Please make explain the reason for your answer using complete sentences.

6. Let $S$ be the set of polynomials $\{1 - 2t + t^2, t - 2t^2 + t^3, 1 - 3t + 3t^2 - t^3\}$. Determine whether or not $S$ is a linearly independent set. **Hint:** Start by finding the coordinate vectors of the polynomials; remember that the degree three polynomial $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ has coordinate vector $\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$. Then use the fact the polynomials in the set are linearly independent if and only if their coordinate vectors are linearly independent.

7. The first four Hermite polynomials are the set $\{1, 2t, -2 + 4t^2, -12t + 8t^3\}$. These polynomials arise naturally in the study of certain important differential equations in mathematical physics. Show that the first four Hermite polynomials form a basis of $\mathbb{P}_3$. **Hint:** The polynomials form a basis of $\mathbb{P}_3$ if and only if their coordinate vectors form a basis of $\mathbb{R}^4$. 