1. If the given statement is true, write “True”. If the given statement is false, write “False” and explain why it is false using complete sentences with proper grammar and punctuation. Giving an example is often a great way to demonstrate that a statement is false!

(a) If $A\vec{x} = \lambda \vec{x}$ for some vector $\vec{x}$, then $\lambda$ is an eigenvalue of $A$.

(b) If $\vec{v}$ and $\vec{w}$ are linearly independent eigenvectors of a matrix $A$, then $\vec{v}$ and $\vec{w}$ must correspond to distinct eigenvalues.

(c) A matrix $B$ is not invertible if and only if 0 is an eigenvalue of $B$.

(d) A number $c$ is an eigenvalue of $A$ if and only if the equation $(A - cI)\vec{x} = \vec{0}$ has a nontrivial solution.

(e) If $B$ is an echelon form of a matrix $A$, then $A$ and $B$ have the same eigenvalues.
2. Construct an example of a $2 \times 2$ matrix with only one distinct eigenvalue.

3. Suppose $A$ is an invertible matrix and suppose that $\lambda$ is one of its eigenvalues. Show that $\lambda^{-1}$ is an eigenvalue of $A^{-1}$.

4. Show that if $A^2$ is the zero matrix, then the only eigenvalue of $A$ is zero.

5. Show that if $A$ and $B$ are similar matrices, then $\det A = \det B$. 
6. If the given statement is true, write “True”. If the given statement is false, write “False” and explain why it is false using complete sentences with proper grammar and punctuation. Giving an example is often a great way to demonstrate that a statement is false!

(a) Let $A$ be an $n \times n$ matrix. If $\mathbb{R}^n$ has a basis of eigenvectors for $A$, then $A$ is diagonalizable.

(b) If a square matrix is invertible, then it is also diagonalizable.

(c) An $n \times n$ matrix is diagonalizable if it has $n$ distinct eigenvectors.

(d) If an $n \times n$ matrix is diagonalizable, then it has $n$ distinct eigenvalues.

(e) If a square matrix is diagonalizable, then it is also invertible.
7. Suppose $A$ is a $5 \times 5$ matrix with two distinct eigenvalues. Suppose also that the eigenspace for one of the eigenvalues is three-dimensional and that the eigenspace for the other eigenvalue is two-dimensional. Is $A$ diagonalizable? Please explain the reason for your answer.

8. Suppose $A$ is a $3 \times 3$ matrix with two distinct eigenvalues and that the eigenspace for each of the two eigenvalues is one-dimensional. Is $A$ diagonalizable? Please explain the reason for your answer.

9. Suppose $A$ is a $4 \times 4$ matrix with three distinct eigenvalues. Suppose also that the eigenspace for one of the eigenvalues is one-dimensional and that the eigenspace for one of the other eigenvalues is two-dimensional. Is it possible that $A$ is not diagonalizable? Please explain the reason for your answer.