Formula Sheet for Second Midterm Exam

Trig identities:

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Integrals of trig functions:

$$\int \sec u\, du = \ln|\sec u + \tan u| + C \qquad \int \csc u\, du = \ln|\csc u - \cot u| + C$$

Average value: Let f be continuous on $[a,b]$. The average value of f on $[a,b]$ is

$$f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)\, dx.$$

Area under a polar curve: Let $\mathcal{R}$ be the region bounded by the polar curve $r = f(\theta)$ and the rays $\theta = a$ and $\theta = b$. Then

$$\text{Area } \mathcal{R} \; = \; \int_a^b \frac{1}{2}[f(\theta)]^2\, d\theta.$$

Complex numbers

- nth powers: If $z = r(\cos\theta + i\sin\theta)$, then for any integer n,

$$z^n = r^n(\cos(n\theta) + i\sin(n\theta)).$$

- nth roots: For any integer n, the nth roots of $r(\cos\theta + i\sin\theta)$ are

$$r^{\frac{1}{n}}\left(\cos\left(\frac{\theta + k2\pi}{n}\right) + i\sin\left(\frac{\theta + k2\pi}{n}\right)\right), \qquad k = 0, 1 \ldots, n-1.$$

- trig functions and complex exponentials:

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

Approximate integration

- Midpoint rule:

$$\int_a^b f(x)dx \approx M_n = \frac{b-a}{n}\sum_{i=1}^{n} f(\bar{x}_i), \quad \bar{x}_i = \text{ midpoint of } [x_{i-1}, x_i]$$

- Trapezoid rule:
$$\int_a^b f(x)dx \approx T_n = \frac{1}{2}(R_n + L_n),$$
where R_n and L_n are, respectively, the right endpoint and left endpoint approximations.
- Simpson's rule:
$$\int_a^b f(x)dx \approx S_{2n} = \frac{1}{3}T_n + \frac{2}{3}M_n.$$

Convergence tests for infinite series:

- n^{th} **Term Test:** If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ diverges.
- **Integral Test:** Suppose f is a continuous, positive, decreasing function on $[k, \infty)$ and let $a_n = f(n)$. Then $\sum_{n=k}^{\infty} a_n$ converges if and only if $\int_k^{\infty} f(x)dx$ converges.
- **Comparison Test:** Suppose $0 \leq a_n \leq b_n$ for all n. If $\sum b_n$ converges, then $\sum a_n$ converges. If $\sum a_n$ diverges, then $\sum b_n$ diverges.
- **Limit Comparison Test:** Suppose $a_n > 0$ and $b_n > 0$ for all n. If $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n}$ exists and is non-zero, then $\sum a_n$ and $\sum b_n$ either both converge or both diverge.
- **Alternating Series Test:** Suppose $\{a_n\}$ is a decreasing positive sequence with $\lim\limits_{n\to\infty} a_n = 0$. Then $\sum(-1)^n a_n$ converges.
- **Ratio Test:** Let $L = \lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right|$. If $L < 1$, then $\sum a_n$ converges. If $L > 1$, then $\sum a_n$ diverges. If $L = 1$, the test is inconclusive.
- **Root Test:** Let $L = \lim\limits_{n\to\infty} |a_n|^{1/n}$. If $L < 1$, then $\sum a_n$ converges. If $L > 1$, then $\sum a_n$ diverges. If $L = 1$, the test is inconclusive.