MATH 180B Homework 1

January 11, 2019

Due Thursday January 17 11:59pm. Pleaes submit your homework in pdf format on Gradescope. Justify your answers to get full credits.

1. Roll an even dice and observe the number N on the uppermost face. Then toss a fair coin N times and observe X, the total number of heads that appear in N tosses.

(i) Write down the conditional probability mass function $p_{X|N}(\cdot|3)$.

(ii) What is $\mathbb{P}(X = 5)$?

(iii) What is $\mathbb{E}(X)$?

2. Suppose U and V are independent geometric random variables with parameter p. Let $Z = U + V$. Determine the conditional probability mass function of $p_{U|Z}(\cdot|n)$ of U given that $Z = n$.

3. Let X be a Poisson random variable with parameter λ. Calculate the conditional expectation of X given that X is odd.

4. Dice #1 is rolled a single time. Dice #2 is rolled repeatedly. The game stops at the first time that the sum of the two dices is 4 or 7. What is the probability that the game stops with a sum of 4?

5. Let N be a Poisson random variable with parameter λ. Suppose $\xi_1, \xi_2, ...$ is a sequence of i.i.d. random variables with mean μ and variance σ^2, independent of N. Let $S_N = \xi_1 + \ldots \xi_N$. Determine the mean and variance of S_N.

6. Let X, Y be independent random variables, each having Exponential(λ) distribution. What is the conditional density function of X given that $Z = X + Y = z$?

7. Suppose the random variable U has uniform distribution on $[0, 1]$. Then a second random variable T is chosen to have uniform distribution on $[0, U]$. Calculate $\mathbb{P}(T > 1/2)$.

8. Let U be uniformly distributed on $[0, L]$, where L has Exponential(λ) distribution. Let $V = L - U$. What is the joint density function of U and V?