1. Roll an even dice and observe the number $N$ on the uppermost face. Then toss a fair coin $N$ times and observe $X$, the total number of heads that appear in $N$ tosses.

(i) Write down the conditional probability mass function $p_{X|N}(\cdot|3)$.

(ii) What is $\Pr(X = 5)$?

(iii) What is $\mathbb{E}(X)$?

2. Suppose $U$ and $V$ are independent geometric random variables with parameter $p$. Let $Z = U + V$. Determine the conditional probability mass function of $p_{U|Z}(\cdot|n)$ of $U$ given that $Z = n$.

3. Let $X$ be a Poisson random variable with parameter $\lambda$. Calculate the conditional expectation of $X$ given that $X$ is odd.

4. Dice #1 is rolled a single time. Dice #2 is rolled repeatedly. The game stops at the first time that the sum of the two dices is 4 or 7. What is the probability that the game stops with a sum of 4?

5. Let $N$ be a Poisson random variable with parameter $\lambda$. Suppose $\xi_1, \xi_2, \ldots$ is a sequence of i.i.d. random variables with mean $\mu$ and variance $\sigma^2$, independent of $N$. Let $S_N = \xi_1 + \ldots + \xi_N$. Determine the mean and variance of $S_N$.

6. Let $X, Y$ be independent random variables, each having Exponential($\lambda$) distribution. What is the conditional density function of $X$ given that $Z = X + Y = z$?

7. Suppose the random variable $U$ has uniform distribution on $[0, 1]$. Then a second random variable $T$ is chosen to have uniform distribution on $[0, U]$. Calculate $\Pr(T > 1/2)$.

8. Let $U$ be uniformly distributed on $[0, L]$, where $L$ has Exponential($\lambda$) distribution. Let $V = L - U$. What is the joint density function of $U$ and $V$?