Homework 9 Math280A Fall 2017

Due Wednesday in class, Dec 6. Relevant sections in Durrett’s textbook 2.5 2.6; in Resnick book: Chapter 7. Justify all your answers.

1. Let $X_1, X_2, \ldots$ be i.i.d. random variables with $E X_1 = 0$ and $E |X_1| < \infty$. Suppose $(c_n)$ is a bounded sequence of real numbers. Show that

$$\frac{1}{n} \sum_{j=1}^{n} c_j X_j \to 0 \text{ a.s.}$$

2. Let $X_1, X_2, \ldots$ be independent random variables. Let $S_n = X_1 + \ldots + X_n$, $S_{m,n} = S_n - S_m$ where $m \leq n$.

(i) Show that

$$P (|S_{m,n}| \geq a) \geq P \left( \max_{m < j \leq n} |S_{m,j}| \geq 2a \right) \cdot \min_{m < k \leq n} P (|S_{k,n}| \leq a).$$

(ii) Use (i) to show the following: if $S_n \to W$ in probability, where $W$ is the limiting random variable, then $S_n$ also converges to $W$ a.s. (Hint: show that $(S_n)$ is a Cauchy sequence a.s.)

3. Let $X_1, X_2, \ldots$ be i.i.d. Poisson random variables with mean 1. Find

$$\lim_{n \to \infty} \frac{1}{n} \log P (S_n \geq na)$$

for $a > 1$, where $S_n = X_1 + \ldots + X_n$.

4. Suppose $X$ is a random variable such that

$$\varphi(t) = E e^{tX} < \infty \text{ for all } t \in \mathbb{R}.$$ 

Use Hölder inequality and Fatou’s lemma to show that $\log \varphi$ is convex and lower semicontinuous on $\mathbb{R}$. 