

# Math 140A: “Winter” 2016

## Homework 10

Available	Friday, March 4	Due	Friday, March 11
-----------	-----------------	-----	------------------

Turn in the homework by 5:00pm on Friday, March 11, in the homework box in the basement of AP&M. Late homework will not be accepted.

1. Let  $\{K_n : n \in \mathbb{N}\}$  be a collection of compact sets. Show that  $\bigcap_n K_n$  is compact.
2. Exercise 1, p. 98 in Rudin.
3. Exercise 4, p. 98 in Rudin.
4. Let  $X, Y$  be metric spaces and let  $f: X \rightarrow Y$  be *uniformly* continuous. If  $(x_n)$  is a Cauchy sequence in  $X$ , show that  $(f(x_n))$  is a Cauchy sequence in  $Y$ . On the other hand, if the assumption of *uniform* continuity is dropped, the result is false: give an example of a continuous function  $f: (0, 1) \rightarrow \mathbb{R}$  that does not map Cauchy sequences to Cauchy sequences.