Math 140A: "Winter" 2016
Homework 2

Available	Friday, January 8	Due	Friday, January 15

Turn in the homework by 5:00pm on Friday, January 15, in the homework box in the basement of AP&M. Late homework will not be accepted.

1. Exercise 1.6, p. 22 in Rudin.

2. Exercise 1.9, p. 22 in Rudin.

3. Let $\mathbb{F}$ be an ordered field, and suppose $\mathbb{F}$ has the nested intervals property: if $a_n, b_n \in \mathbb{F}$ satisfy

$$a_1 \le a_2 \le \cdots \le a_n \le \cdots \le b_n \le \cdots \le b_2 \le b_1$$

and $b_n - a_n < \frac{1}{n}$ for each $n \in \mathbb{N}$, then $\bigcap_{n\in\mathbb{N}}[a_n, b_n]$ is nonempty, and consists of exactly one point. Prove that $\mathbb{F}$ is a complete ordered field.

4. Let $\mathbb{F}$ be a complete ordered field. Let $M \in \mathbb{F}$. For each n, let $a_n \in \mathbb{F}$ and $b_n \in \mathbb{F}$ satisfy $a_0 \le a_1 \le \cdots \le a_n \le \cdots \le M$ and $b_0 \le b_1 \le \cdots \le b_n \le \cdots \le M$. Prove that

$$\sup\{a_n + b_n \colon n \in \mathbb{N}\} = \sup\{a_n \colon n \in \mathbb{N}\} + \sup\{b_n \colon n \in \mathbb{N}\}. \qquad (*)$$

On the other hand, without the assumption that a_n and b_n are increasing, provide a counterexample to $(*)$.