Math 140A: "Winter" 2016
Homework 4

Available	Friday, January 22	Due	Friday, January 29

Turn in the homework by 5:00pm on Friday, January 29, in the homework box in the basement of AP&M. Late homework will not be accepted.

Throughout this homework set, assume all sequences are in $\mathbb{R}$.

1. Suppose $a_n \geq 0$ for all sufficiently large n. Show that $\limsup_{n\to\infty} a_n = 0$ if and only iff $\lim_{n\to\infty} a_n = 0$. Does the same hold for $\liminf_{n\to\infty} a_n$?

2. Let (a_n) and (b_n) be bounded sequences.

(a) Show that
$$\limsup_{n\to\infty}(a_n + b_n) \leq \limsup_{n\to\infty} a_n + \limsup_{n\to\infty} b_n.$$

(b) Suppose that $a_n \geq 0$ and $b_n \geq 0$ for all sufficiently large n. Show that
$$\limsup_{n\to\infty}(a_n \cdot b_n) \leq \limsup_{n\to\infty} a_n \cdot \limsup_{n\to\infty} b_n.$$

3. Let (a_n) be a bounded sequence. Show that $\overline{a} = \limsup\limits_{n\to\infty} a_n$ and $\underline{a} = \liminf\limits_{n\to\infty} a_n$ are the unique number with the following properties: for all $\epsilon > 0$,
$$\begin{aligned} &a_n \leq \overline{a} + \epsilon \text{ for all sufficiently large } n\text{, and} \\ &a_n \geq \overline{a} - \epsilon \text{ for infinitely many } n, \end{aligned}$$
and
$$\begin{aligned} &a_n \leq \underline{a} + \epsilon \text{ for infinitely many } n\text{, and} \\ &a_n \geq \underline{a} - \epsilon \text{ for all sufficiently large } n. \end{aligned}$$

4. Let (a_n) be a bounded sequence with the following property: all convergent subsequences of (a_n) converge to the same limit a. Prove that $a_n \to a$.