Math 140A: "Winter" 2016
Homework 7

Available	Friday, February 12	Due	Friday, February 19

Turn in the homework by 5:00pm on Friday, February 19, in the homework box in the basement of AP&M. Late homework will not be accepted.

1. Let (a_n) and (b_n) be two sequences of non-negative real numbers. Suppose $a = \lim_{n\to\infty} a_n$ exists, and $a > 0$. Show that

$$\limsup_{n\to\infty} a_n b_n = a \cdot \limsup_{n\to\infty} b_n.$$

2. Exercise 3.8, p. 79 in Rudin.

3. Exercise 3.11, p. 79 in Rudin.

4. (Weak Stirling's approximation) Let $\alpha < e$. Prove that there is a constant $C > 0$ (which may depend on α) so that

$$n! < C \cdot \left(\frac{n}{\alpha}\right)^n, \qquad \forall\, n \in \mathbb{N}.$$

[*Hint*: For any finite N, this holds with some $C = C_N$ for all $n \le N$. Use the fact that the sequence $(1 + 1/n)^n$ is increasing and converges to e to choose N appropriately in terms of α. Then proceed by induction.]

Stirling's approximation says that

$$\sqrt{2\pi} \cdot \sqrt{n} \cdot \left(\frac{n}{e}\right)^n \le n! \le e \cdot \sqrt{n} \cdot \left(\frac{n}{e}\right)^n, \qquad \forall\, n \in \mathbb{N}.$$

Exercise 4 comes close to proving the upper bound; the lower bound shows that we *cannot* let $\alpha = e$ in the proof, since there is an additional $\sqrt{n}$ factor which, although much smaller than n^n, still blows up as $n \to \infty$.