# MATH 140A: FOUNDATIONS OF REAL ANALYSIS I
# SUMMARY OF KEY FACTS FOR THE FINAL EXAM

TODD KEMP

Throughout, reference numbers (like Definition 6.1 and Proposition 5.30) refer to the course lecture notes, also available on the website.

## Topics *after* Exam 2

(1) Compact sets are closed; closed subsets of compact sets are compact (Proposition 5.27)
(2) Compact sets are bounded (Proposition 5.30)
(3) The Heine–Borel Theorem: in Euclidean space $\mathbb{R}^m$, a set is compact iff it is closed and bounded (Theorem 5.31); this is *not* true in general (Example 5.32)
(4) Nested compact sets have non-empty intersection (Proposition 5.33)
(5) The Cantor set (Example 5.34)
(6) Limits of functions, defined through limits of sequences (Definition 6.1)
(7) Limit theorems for $\mathbb{R}$- or $\mathbb{C}$-valued functions (Theorem 6.5)
(8) The $\epsilon$-$\delta$ definition of limits of functions (Theorem 6.6)
(9) Continuity (Definition 6.8)
(10) Continuity vs. limits: need not remove the base-point for the limit when proving continuity (Proposition 6.9)
(11) Some fun pathological examples of discontinuous functions (Examples 6.11 and 6.12)
(12) Uniform continuity (Definition 6.14)
(13) All continuous functions on compact domains are uniformly continuous (Theorem 6.17)
(14) The continuous image of a compact set is compact (Proposition 6.18)
(15) Any continuous real-valued function on a compact set achieves its maximum and minimum (Corollary 6.19)

## Some Useful Hints.

(1) When proving a limit of a function exists, it is often easier to use the *sequence* definition of limit instead of the $\epsilon$-$\delta$ definition – it is (psychologically, anyhow) easier to find a time $N$ after which things happen (on a discrete set of indices $n \geq N$) than to find a real number $\delta$ so that things happen in a $\delta$-ball around the base-point. When proving a limit does not exist, either definition works well, and they amount to exactly the same thing: find some point inside a small ball that gets maps far away from the target.

(2) On the other hand, in order to prove uniform continuity requires the $\epsilon$-$\delta$ definition: you must be able to show that the $\delta$ can be chosen independently of the base-point. See, however, Theorem 6.17 for a powerful tool in proving uniform continuity; Exercise 4 on Homework 10 can also be useful in proving that a function is *not* uniformly continuous.

*Date*: March 12, 2016.

(3) General advice: learn (well) all the definitions and terminology from this course. It will serve you well not only on the Final Exam, but also in all subsequent mathematics courses you take.

(4) Study advice: rework homework problems, the midterm problems, and the practice problems. Sleep and eat normally: pulling an all-nighter before the exam will *not* help. And try to remember, even when you feel challenged: the best way to approach the problems on the exam is to have fun with them.