Math 140B: Spring 2016
Homework 5

Available Friday, April 22 || Due Friday, April 29

Turn in the homework by 5:00pm in the homework box in the basement of AP&M. Late homework will not be accepted.

1. Exercise 26, p. 118 in Rudin.

2. Assume $f : (0, 2) \rightarrow \mathbb{R}$ is differentiable, and that there is a constant $M > 0$ so that $|f'(x)| \leq M$ for all $x \in (0, 2)$. Let $a_n = f(1/n)$ for $n \in \mathbb{N}$. Prove that $(a_n)$ is a convergent sequence.

3. Take for granted that $\sin$ and $\cos$ are differentiable functions on $\mathbb{R}$ satisfying $\sin' = \cos$ and $\cos' = -\sin$. Show that $\sin x \approx x$ is a good approximation for small $x$ by using Taylor’s theorem to obtain

$$|\sin x - x| \leq \frac{1}{6}|x|^3 \quad \forall x \in \mathbb{R}.$$

Use this to calculate the limit for different values of $a \in \mathbb{R}$ and $c > 0$ of the function $f(t) = t^a \sin(|t|^{-c})$ (of Exercise 13 on p. 115 in Rudin), as $t \to \infty$.