This is a one-hour exam, but you may take up to two hours. You may bring one double-sided sheet of notes. No calculators or other electronic devices, or other written materials are allowed during the exam. Point values (totaling 40) are indicated for each problem. Please do all your work in a Blue Book from the UCSD bookstore; make sure to write your name and student number clearly on each Blue Book you turn in. Have fun!

1. (10 points) Let $S$ be a connected set in a metric space. Prove that its closure $\overline{S}$ is connected.

2. (a) (5 points) Give an example of a function $f: \mathbb{R} \to \mathbb{R}$ and an open set $U \subseteq \mathbb{R}$ such that $f(U)$ is closed.

   (b) (5 points) Suppose $f: \mathbb{R} \to \mathbb{R}$ is $C^1$, and maps every open set to a closed set. Prove that $f$ is constant.

3. (10 points) [For this exercise, you may freely use the fact that $\sin$ is a differentiable function whose derivative is $\sin' = \cos$.] Let $f(x) = x + \frac{1}{2} \sin x$. Prove that

   \[
   \frac{1}{2} |x - y| \leq |f(x) - f(y)| \leq \frac{3}{2} |x - y| \quad \text{for all } x, y \in \mathbb{R}.
   \]

4. Let $f: (-1, 1) \to \mathbb{R}$ be a function for which $f'(0)$ exists.

   (a) (5 points) Suppose that $f\left(\frac{1}{n}\right) = 0$ for all $n \geq 1$. Prove that $f(0) = f'(0) = 0$.

   (b) (5 points) Prove that $\lim_{t \to 0} \frac{f(t) + f(-t)}{t} = 0$. 
