Today: Midterm 1 Review

Next: §2.2-2.3: Matrix Inverse

Midterm 1: TONIGHT !

Beginning @ 8pm in GH 242, York 2622 & York 2722

check TritonEd for your room/seat

- One double-sided 8.5"x 11" note sheet
- No electronic devices
- Write answers on exam booklet. We will provide scratch paper if needed.
- Bring UCSD Student ID.
- Have fun tonight!

Math 18: Lecture C **Midterm Exam 1 v. B** **Name:** ____________
February 1, 2017 **(Total Points: *30*)** **PID:** ____________

Instructions

1. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
2. **You may use one handwritten, double-sided page of notes, but no books or other assistance during this exam.**
3. **Read each question carefully and answer each question completely.**
4. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
5. **Write your Name at the top of each page.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Consider the following system of equations.

$$\begin{aligned} x_1 - 2x_2 &= 0 \\ 3x_1 + hx_2 &= 0 \\ x_1 + 2x_2 &= 4 \end{aligned}$$

(a) Show that there is a unique value of h for which the system is consistent, and find that value of h.

(b) In the case that the system is consistent, does it have a unique solution, or infinitely many solutions? Justify your answer.

(9 points) 2.

The matrix $A = \begin{bmatrix} 1 & -3 & 3 & 7 \\ -3 & 7 & -1 & -11 \\ 0 & 1 & -4 & -5 \end{bmatrix}$ has reduced row-echelon form $\begin{bmatrix} 1 & 0 & -9 & -8 \\ 0 & 1 & -4 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Describe the general solution of the system of equations whose augmented matrix is A.

(b) Describe the general solution to the vector equation $x_1 \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 7 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(c) Let C be the 3×3 matrix given by the first three columns of A. Is the system $C\mathbf{x} = \mathbf{b}$ consistent for all possible choices of $\mathbf{b} \in \mathbb{R}^3$? Briefly explain your answer.

(6 points) 3. Let A be any 7×12 matrix. For each statement about A, circle **T** if it is *always* True; circle **F** if it is *ever* False. 2 points will be assigned for each correct response, 1 point for each blank non-response, and 0 points for each incorrect response. No justification is required.

(T F) The columns of A span $\mathbb{R}^7$.

(T F) The columns of A are linearly *de*pendent.

(T F) The matrix equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$.

(8 points) 4. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be the following three vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ -3 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 8 \\ 5 \\ -1 \end{bmatrix}.$$

(a) Determine if the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent. If they are not, exhibit a non-trivial linear combination of them that yields the $\mathbf{0}$ vector.

(b) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation satisfying

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \mathbf{v}_1, \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \mathbf{v}_2, \quad T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \mathbf{v}_3.$$

Is T one-to-one? Justify your answer.

Math 18 **Midterm Exam 1 v. B** **Name:** ____________________
February 1, 2017 **(Total Points: 25)** **PID:** ____________________

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(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Consider the following system of linear equations.

$$\begin{array}{rcrcl} x_1 & + & 2x_2 & = & 1 \\ 2x_1 & + & x_2 & = & 4 \\ x_1 & - & x_2 & = & h \end{array}$$

(a) Find all value(s) of h for which the system is consistent, and describe the corresponding solution set.

(b) Find all value(s) of h for which the system is *inconsistent*.

(c) Is the corresponding homogeneous system consistent? If so, describe its solution set.

(6 points) 2. Let $A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 2 \\ 3 & 1 & 5 \end{bmatrix}$.

(a) Find the reduced row echelon form of A.

(b) Describe the solution set of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

(c) Let $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Is the equation $A\mathbf{x} = \mathbf{b}$ consistent? If it is, describe the solution set.

(6 points) 3. For each $k \in \mathbb{R}$, let S_k be the set of vectors in $\mathbb{R}^3$ given by $S_k = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ k \end{bmatrix} \right\}$.

For each of parts (a) - (c), find the value(s) of k for which S_k has the indicated property. Be sure to show how you arrived at each answer.

(a) S_k is linearly independent.

(b) S_k is linearly dependent.

(c) S_k spans $\mathbb{R}^3$.

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(6 points) 4. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}\right) + T\left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right) + T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Is T one-to-one? Justify your answer.