Today: § 2.2-2.3: Inverse Matrix
& § 4.1: Vector Spaces & Subspaces
Next: § 4.2: Null Spaces & Column Spaces

Homework:
MATLAB Assignment #3: Due Feb 9 (Friday)
MyMathLab Homework #4: Due Feb 12 (Monday)
Recap: A matrix $A$ is invertible if there is a matrix $C$ with $CA = I = AC$. We denote $C = A^{-1}$ in this case.

* Only square matrices can be invertible.
* $A$ is invertible if and only if $\text{rref} (A) = I$
  i.e. every row of $A$ is pivotal
  i.e. every column of $A$ is pivotal
  i.e. the columns of $A$ are linearly independent and span $\mathbb{R}^n$.
* To find $A^{-1}$, we use row reduction:

$$\begin{bmatrix} A & I \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} I & A^{-1} \end{bmatrix}$$
E.g. General 2×2 matrix

\[
\begin{bmatrix}
a & b & 1 & 0 \\
c & d & 0 & 1 \\
\end{bmatrix}
\]

A

Theorem: A is invertible iff

and in this case

E.g. \[
\begin{bmatrix}
1 & 2 \\
3 & 4 \\
\end{bmatrix}
\]

E.g. \[
\begin{bmatrix}
1 & 2 \\
1 & 2 \\
\end{bmatrix}
\]
Properties of inverses:

\((A^{-1})^{-1}\)

\((AB)^{-1}\)

\((A^T)^{-1}\)
§ 4.1: Vector Spaces

A "vector space" is a set of objects (which we call vectors) on which two operations are defined:

addition: \( u, v \mapsto u + v \)

scalar multiplication: \( \lambda, v \mapsto \lambda v \) for \( \lambda \in \mathbb{R} \)

Eg. \( \mathbb{R}^n \)
More precisely: "\( V \) is a vector space" means \( V \) is a set, with a distinguished element called \( 0 \), and two operations defined on it, \( + \) and \( \cdot \), which together satisfy the zillion properties on p. 192 in the textbook.
E.g. $P$
E.g. $P_3$ (aka $P_{S_3}$)
$E_g. \left\{ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} : x, y \in \mathbb{R} \right\}$

$E_g. \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$
**Definition**: If $V$ is a vector space, and $W \subseteq V$ is nonempty, $W$ a subspace of $V$ if $W$ is closed under addition and scalar multiplication (in $V$).

E.g., $W = \{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \}$ is a subspace of $\mathbb{R}^3$.

E.g., $S^1 \subset \mathbb{R}^2$
The example \[ \{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \} \] can be re-expressed as

\[ \{ x \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \} = \text{Span} \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \]

More general example: Let \( V \) be a vector space, and consider two vectors \( u \) and \( v \) in \( V \). Then

is a subspace of \( V \).