Today: § 2.2-2.3: Inverse Matrix

& § 4.1 : Vector Spaces & Subspaces

Next: § 4.2: Null Spaces & Column Spaces

Homework:

MATLAB Assigment #3: Due Feb 9 (Friday)

MyMathLab Homework #4: Due Feb 12 (Monday)

Recap: A matrix A is invertible if there is a matrix C with $CA = I = AC$.
We denote $C = A^{-1}$ in this case.

* Only square matrices can be invertible.
* A is invertible if and only if $\mathrm{rref}(A) = I$
  i.e. every row of A is pivotal
  i.e. every column of A is pivotal
  i.e. the columns of A are linearly independent and span $\mathbb{R}^n$.
* To find A^{-1}, we use row reduction:

$$\left[\, A \;\vdots\; I \,\right] \xrightarrow{\mathrm{rref}} \left[\, I \;\vdots\; A^{-1} \,\right]$$

E.g. General 2×2 matrix

$$\left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right]$$

A

Theorem: A is invertible iff

and in this case

E.g. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

E.g. $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

Properties of inverses:

$(A^{-1})^{-1}$

$(AB)^{-1}$

$(A^T)^{-1}$

§ 4.1 : Vector Spaces

A "vector space" is a set of objects (which we call vectors) on which two operations are defined:

addition : $u, v \longmapsto u+v$

scalar multiplication : $\lambda, v \longmapsto \lambda v$ for $\lambda \in \mathbb{R}$

E.g. $\mathbb{R}^n$

More precisely: "V is a vector space" means V is a set, with a distinguished element called $\underline{0}$, and two operations defined on it, $+$ & $\cdot$, which together satisfy the zillion properties on p. 192 in the textbook.

E.g. $\mathbb{P}$

E.g. $\mathbb{P}_{=3}$

Eg. $\mathbb{P}_3$ (aka $\mathbb{P}_{\leq 3}$)

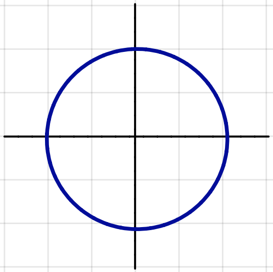
Eg. $\left\{ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} : x, y \in \mathbb{R} \right\}$

Eg. $\left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$

Definition: If V is a vector space, and $W \subseteq V$ is nonempty, W a **subspace** of V if W is closed under addition and scalar multiplication (in V).

E.g. $W = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^3$.

E.g. $S^1 \subset \mathbb{R}^2$



The example $\left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$ can be re-expressed as

$$\left\{ x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\} = \operatorname{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

More general example: Let V be a vector space, and consider two vectors $\mathbf{u}$ and $\mathbf{v}$ in V. Then

is a subspace of V.