Today: § 4.1: Vector Spaces & Subspaces

& § 4.2: Null Spaces & Column Spaces

Next: § 4.3: Linear Independent Sets; Bases

Homework:

MATLAB Assigment #3: Due Feb 9 (Friday)

MyMathLab Homework #4: Due Feb 12 (Monday)

A vector space is a set V

together with a special element 0

and equipped with two operations

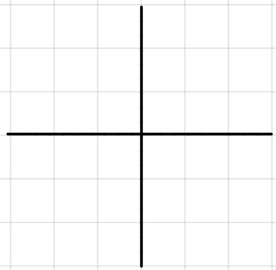
$+$ & scalar mult.

satisfying all the usual rules.

Eg. $M_{2\times 3} = \{ 2\times 3 \text{ matrices} \}$ is a vector space.

Eg. The line $L = \left\{ t\begin{bmatrix} 1 \\ 2 \end{bmatrix} : t \in \mathbb{R} \right\}$

is a vector space



Definition: If V is a vector space, and $W \subseteq V$ is nonempty, W a subspace of V if W is closed under addition and scalar multiplication (in V).

That is to say: W is closed under addition (in V)

& W is closed under scalar multiplication (in V)

& $\underline{0}$ (the zero vector in V) is in W.

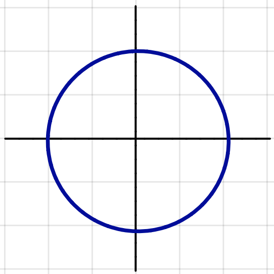
E.g. $\{\underline{0}\}$ ← the trivial subspace.

Eg. $W = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} : x, y \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^3$.

Eg. $A = \left\{ \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} : x, y \in \mathbb{R} \right\}$

Eg. The set $\left\{ \begin{bmatrix} a & b & 0 \\ c & 0 & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\} \subseteq M_{2\times 3}$

Eg. $S^1 \subset \mathbb{R}^2$



E.g. Let V be any vector space, and let $\underline{v}_1, \dots, \underline{v}_n$ be vectors in V. Then $\text{span}\{\underline{v}_1, \dots, \underline{v}_n\}$ is a subspace of V.

E.g. $\left\{ \begin{bmatrix} a-2b \\ 2a+b \\ b \end{bmatrix} : a, b \in \mathbb{R} \right\}$

§4.2: Null Space & Column Space

Given any matrix $A \in M_{m\times n}$ there are two important subspaces:

The null space of A, $Nul(A)$

The column space of A, $Col(A)$

We will see shortly that they are subspaces – but of what?

Theorem: For $A \in M_{m\times n}$, $\mathrm{Nul}(A)$ is a subspace of $\mathbb{R}^n$
& $\mathrm{Col}(A)$ is a subspace of $\mathbb{R}^m$

E.g. $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

$\operatorname{rref}(A) = \begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Eg. $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$ $\underline{u} = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ $\underline{v} = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$

Q1: is $\underline{u} \in \text{Nul}(A)$?

Q2: is $\underline{v} \in \text{Col}(A)$?