Today: § 4.4: Coordinates

& § 3.1-3.2: Determinants

Next: § 3.2-3.3: Determinants & Volume

Reminders:

MyMathLab Homework #5: Due Tuesday by 11:59pm

MATLAB Assigment #4: Due February 23 by 11:59pm

Midterm #2: Feb 28, 8-10pm

Reminder: If V is a vector space and $\mathcal{B} = \{\underline{b}_1, \underline{b}_2, \dots, \underline{b}_n\}$ is a basis for V, the **coordinate vector** $[\underline{v}]_{\mathcal{B}}$ of any vector $\underline{v} \in V$ is the column of coefficients in the unique expansion of $\underline{v}$ in the basis $\mathcal{B}$:

$$\underline{v} = x_1 \underline{b}_1 + x_2 \underline{b}_2 + \dots + x_n \underline{b}_n \rightsquigarrow [\underline{v}]_{\mathcal{B}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Theorem: If $\mathcal{B} = \{\underline{b}_1, \underline{b}_2, \dots, \underline{b}_n\}$ is a basis for V, then the function $T : V \to \mathbb{R}^n : T(\underline{v}) = [\underline{v}]_{\mathcal{B}}$ is a one-to-one linear transformation from V onto $\mathbb{R}^n$.

Such a linear transformation is called an

E.g. $A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\left\{ \qquad \qquad \right\} = \mathcal{B}$ basis for $\mathrm{Col}(A)$.

The point of isomorphisms is: they preserve all linear properties.

<u>Theorem</u>: Let $\{\underline{b}_1, \underline{b}_2, \dots, \underline{b}_n\} = \mathcal{B}$ be a basis for V.

Then

* $\{\underline{v}_1, \dots, \underline{v}_k\} \in V$ are linearly independent in V
  iff $[\underline{v}_1]_{\mathcal{B}}, \dots, [\underline{v}_k]_{\mathcal{B}}$ are linearly independent in $\mathbb{R}^n$.
* " " ______ span ______ V
  " " ______ span ______ $\mathbb{R}^n$.

Eg. Show that $\{1, x-1, (x-1)^2\}$ is a basis for $\mathbb{P}_2$.

§3.1 Determinants

If A is a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, when doing row reduction, we find that the columns are both pivotal unless $\underbrace{ad-bc}=0$.

This is the **determinant** of A, denoted $\det A = |A|$.

Theorem: A is invertible iff

in which case $A^{-1} =$

For any $n \times n$ matrix A, there is a quantity $\det A$ which

* is a polynomial function of the entries of A
* determines whether A is invertible.

The 2×2 case has a simple formula; for larger matrices, it is more complicated.

The generalization of the 2×2 case is the **cofactor expansion**.

E.g. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

Cofactor Expansion

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\det A = a_{j1} C_{j1} + a_{j2} C_{j2} + \cdots + a_{jn} C_{jn}$$ for any row

$$= a_{1j} C_{1j} + a_{2j} C_{2j} + \cdots + a_{nj} C_{nj}$$ or column j.

Eg. $\begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & -1 \\ 0 & -2 & 0 \end{bmatrix}$

E.g.

$$A = \begin{bmatrix} 3 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

Theorem: If A is triangular $\det A$ is the product of the diagonal entries.

§ 3.2: Determinants & Row Operations

* $A \xrightarrow[R_i \to R_i/p]{} \tilde{A}$:

* $A \xrightarrow[R_i \leftrightarrow R_j]{} \tilde{A}$:

* $A \xrightarrow[R_i \to aR_1 + R_i]{} \tilde{A}$:

Theorem: If A is $n \times n$ and has full rank (ie. $\operatorname{rank} A = n$ ie. A is invertible) then $\det A = \pm$ product of the pivots in row reduction.

If $\operatorname{rank} A < n$, $\det A = 0$. . .

Eg.

$$\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$