Today: § 3.2-3.3: Determinants

Next: § 5.1: Eigenvalues

<u>Reminders</u>:

MATLAB Assigment #4: Due this Friday by 11:59pm

MyMathLab Homework #6: Due Tuesday by 11:59pm

Midterm #2: 1 week from tonight, 8-10 pm
watch for room/seat assignment
on TritonEd

§ 3.2 Properties of Determinants

The function $\det : M_n \to \mathbb{R}$ is **not** linear.

But it is multilinear. Think of det as a function of the rows:

E.g. $\det \begin{bmatrix} x_1+y_1 & x_2+y_2 & x_3+y_3 \\ 2 & 1 & 4 \\ 3 & -1 & -5 \end{bmatrix}$

Theorem:

Determinants & Row Operations

* $R_i/p \longrightarrow R_i$: $\det\begin{bmatrix} \frac{1}{p}(- & R_1 & -) \\ - & R_2 & - \\ - & R_3 & - \end{bmatrix}$

* $aR_i + R_j \longrightarrow R_j$: $\det\begin{bmatrix} R_1 \\ aR_1 + R_2 \\ R_3 \end{bmatrix} = a\det\begin{bmatrix} R_1 \\ R_1 \\ R_3 \end{bmatrix} + \det\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}$

* $R_i \longleftrightarrow R_j$: $\det\begin{bmatrix} - & R_1 & - \\ - & R_2 & - \\ - & R_3 & - \end{bmatrix}$ $\qquad \det\begin{bmatrix} - & R_2 & - \\ - & R_1 & - \\ - & R_3 & - \end{bmatrix}$

Theorem: If A is $n \times n$ and has full rank

(ie. $\operatorname{rank} A = n$ ie. A is invertible)

then $\det A = \pm$ product of the pivots in row reduction.

If $\operatorname{rank} A < n$, $\det A = 0$.

In particular, $\det A = 0$ if and only if A is not invertible.

Eg. $\begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$

Elementary Matrices

An **elementary matrix** is the result of one row operation applied to the identity matrix.

Eg

Theorem: If r is a row op and $I \xrightarrow{r} E$, then for any matrix A, if $A \xrightarrow{r} \tilde{A}$,

$$\tilde{A} = EA.$$

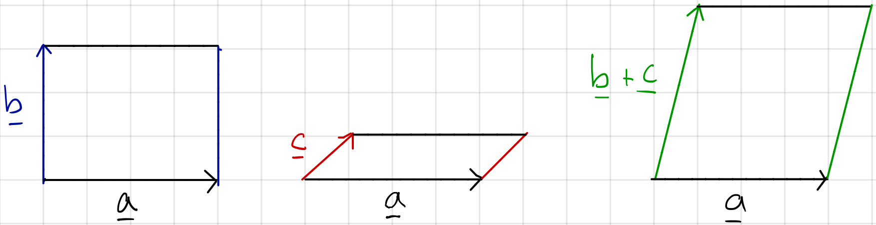
Let A be an invertible matrix. So $\text{rref}(A) = I$.

Theorem: If A, B are $n \times n$ matrices,

$$\det(AB) = \det A \cdot \det B.$$

What **is** the determinant? What does it mean?

Parallelograms:



Theorem: Given two vectors $\underline{a}, \underline{b} \in \mathbb{R}^2$, the area of the parallelogram they determine is

$$A(\underline{a}, \underline{b}) = \left| \det [\underline{a}, \underline{b}] \right|$$

$$= \left| \det \begin{bmatrix} \underline{a}^T \\ \underline{b}^T \end{bmatrix} \right|$$

What does the sign of $\det[\underline{a}, \underline{b}]$ mean?

What about $n \times n$ determinants with $n > 2$?

Parallelepipeds