Today: §3.3 Determinants & §5.1: Eigenvalues

Next: §5.2: Characteristic Polynomial

Reminders:

MATLAB Assignment #4: Due TONIGHT by 11:59pm

MyMathLab Homework #6: Due Tuesday by 11:59pm

Midterm #2: Next Wednesday, 8-10pm
Room/seat assgn. & practice
midterms posted on course webpage.

Properties of Determinants

* Defined by cofactor expansion (recursive expression, expand along any row or column)
* Behaves nicely under row ops: $\det A = \pm$(product of pivots in row reduction to rref(A)).
* $\det A = 0$ iff A is not invertible
* $\det : M_n \to \mathbb{R}$ is multilinear in each row and column of the matrix, with the others held fixed.
* $\det(AB) = \det(A)\det(B)$. $\therefore$ $\det(A^n) = (\det(A))^n$.

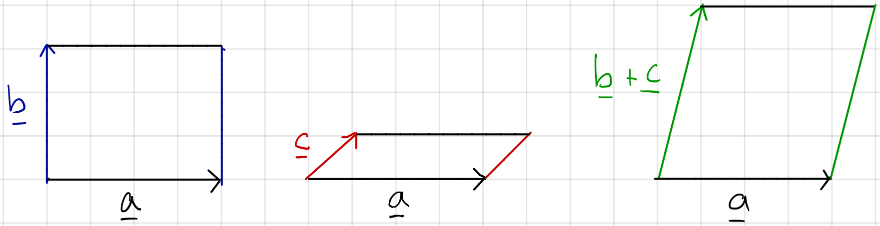
TWO MORE PROPERTIES:

$\det(A^{-1}) =$

$\det(A^T) =$

What $\underline{\underline{\text{is}}}$ the determinant? What does it mean?

Parallelograms:



Theorem: Given two vectors $\underline{a}, \underline{b} \in \mathbb{R}^2$, the area of the parallelogram they determine is

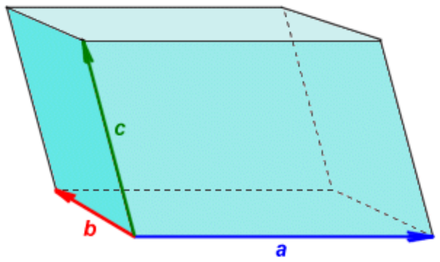
$$A(\underline{a}, \underline{b}) = \left|\det[\underline{a}, \underline{b}]\right|$$

$$= \left|\det\begin{bmatrix} \underline{a}^T \\ \underline{b}^T \end{bmatrix}\right|$$

What does the sign of $\det[\underline{a}, \underline{b}]$ mean?

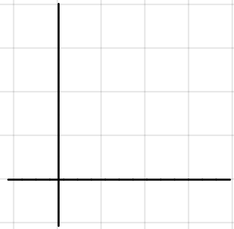
What about $n \times n$ determinants with $n > 2$?

Parallelepipeds



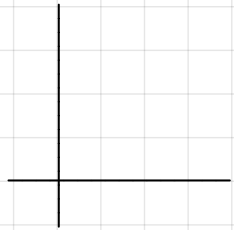
§ 5.1 / A linear transformation $T: V \to V$ tends to move vectors around.

E.g. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x-2y \\ x \end{bmatrix}$



$T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) =$

$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) =$



E.g. If A is a stochastic matrix (row sums = 1) then

Definition: Let $T: V \to V$ be a linear transformation. If there is a non-zero $\underline{v} \in V$ such that

$$T(\underline{v}) = \lambda \underline{v}$$

for some scalar $\lambda \in \mathbb{R}$, we call $\underline{v}$ an eigenvector of T. The scalar λ is the corresponding eigenvalue. (We similarly talk about eigenvectors/eigenvalues of square matrices.)

Eg. Show that 7 is an eigenvalue of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$

Theorem: For any $\lambda \in \mathbb{R}$, the set of vectors $\underline{v}$ for which

$$T(\underline{v}) = \lambda \underline{v}$$

is a subspace of V, called the eigenspace of λ.

(We only call λ an eigenvalue if its eigenspace is $\neq \{\underline{0}\}$.)

Eg. The rotation matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (ccw 90°) has no eigenvalues.

Eg. Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. 2 is an eigenvalue for A; find a basis for the eigenspace.

* Given a (potential) eigenvalue, finding the corresponding eigenspace is routine.
* Finding the eigenvalues is typically hard. (Row reduction changes the eigenvalues.)

One case where eigenvalues can be read off:

Theorem: If A is triangular, its eigenvalues are its diagonal entries.

E.g. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

E.g. $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$