

Today: § 1.2: Row Reduction & Echelon Forms

Next: § 1.3: Vector Equations

Reminders:

MyMathLab Homework #1 & #2: Due Mon, Jan 22

MATLAB Homework #1: Due Fri Jan 19

$$x_1 - 2x_2 + x_3 = 0$$

$$2x_2 - 8x_3 = 8$$

$$5x_1 - 5x_3 = 10$$

↓

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array} \right]$$

"Augmented matrix"

$$\text{Eg.} \quad x_2 - 4x_3 = 8$$

$$2x_1 - 3x_2 + 2x_3 = 1$$

$$4x_1 - 8x_2 + 12x_3 = 1$$

E.g. $\left[\begin{array}{ccc|c} 1 & 5 & 2 & -6 \\ 0 & 4 & -7 & 2 \\ 0 & 0 & 5 & 0 \end{array} \right]$ consistent?

E.g. $2x_1 - x_2 = h$
 $-6x_1 + 3x_2 = k$

For which h, k is the system consistent?

For which h, k is there a unique solution?

Echelon Forms

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 5/2 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 29 \\ 0 & 1 & 0 & 16 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

"leading entry"

Row Echelon Form:

$$\left[\begin{array}{cccccccc|c} 0 & \square & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \square & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \square & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \square & * & * & * \end{array} \right]$$

Reduced Row Echelon Form:

Theorem: By performing row operations, every matrix can be transformed to reduced row echelon form. And that form is unique!

Eg.
$$\begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

In reduced row echelon form, the matrix is

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & -24 \\ 0 & 1 & -2 & 2 & -7 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

i.e. the original system is equivalent to

$$\begin{aligned} x_1 - 2x_3 + 3x_4 &= -24 \\ x_2 - 2x_3 + 2x_4 &= -7 \\ x_5 &= 4 \end{aligned}$$

pivotal columns

pivotal variables

free variables