Today §5.1: Eigenvalues

& §5.2: Characteristic Polynomial

Next: Review for Midterm 2

Reminders:

MyMathLab Homework #6: Due TOMORROW by 11:59pm

Midterm #2: This wednesday 8-10 pm

Room/seat assgn. & practice midterms posted on Course webpage.

Given an $n \times n$ matrix A, an eigenvector is a nonzero vector $\underline{v} \in \mathbb{R}^n$ with the property that

$$A \underline{v} = \lambda \underline{v}$$

for some scalar $\lambda \in \mathbb{R}$, called the eigenvalue.

- The set of eigenvectors of A for a given eigenvalue λ is equal to $\mathrm{Nul}(A - \lambda I)$. So it is a subspace of $\mathbb{R}^n$, called the eigenspace for λ. The eigenvectors are the nonzero vectors in the eigenspace.

- Typically hard to find the eigenvalues of a matrix; once known, finding the eigenspace is routine.

Eg. Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. 2 is an eigenvalue for A; find a basis for the eigenspace.

E.g. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

E.g. $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Theorem. Eigenvectors with distinct eigenvalues are linearly independent.

Corollary: If A is an $n \times n$ matrix with n distinct eigenvalues, then there is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A.

$$[\underline{w}]_{\mathcal{B}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \implies [A\underline{w}]_{\mathcal{B}} = \begin{bmatrix} \lambda_1 x_1 \\ \lambda_2 x_2 \\ \vdots \\ \lambda_n x_n \end{bmatrix} = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \\ & & & \lambda_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

What does that really mean?

Definition: $n \times n$ matrices A and B are called **similar** if there is an invertible $n \times n$ matrix P with the property

$$A = PBP^{-1}$$

- We just saw that, if A has all distinct eigenvalues, then it is similar to a diagonal matrix. That's important; more on that next time.
- Similarity is not the same as row equivalent.

E.g. $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ row equivalent, but not similar.

Theorem: If A and B are similar, then they have the same eigenvalues; and the eigenspaces for A have the same dimensions as the eigenspaces for B.

How can we actually find the eigenvalues?

Definition: The **characteristic polynomial** of a square matrix A is $p_A(\lambda) =$

Theorem: If A is $n \times n$, p_A is a degree n polynomial, whose roots are the eigenvalues of A.

Eg. $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$

Eg. $A = \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}$

Eg. $A = \begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}$